



Evaluation Method for Bending Beam Tests of Fibre Reinforced Concrete

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Abstract. There are several existing guidelines for determining the material parameters of fibre reinforced concrete (FRC). The most commonly used test methods for this determination are the four-point and three-point bending beam tests. Generally, owing to the relatively small size of specimens and the random locations of fibres, the standard deviation of test data is high. Modifying the evaluation method can lead to better approximations of the material parameters without requiring additional experiments. This study proposes a method that considers the number and location of fibres when estimating the post-crack performance of a cracked cross section in three-point notched bending beam tests. The test results of the steel and macro synthetic FRC were evaluated using both Eurocode standards and the proposed method; and the evaluation results were compared. Despite the proposed method being more complex and having an evaluation more effort-intensive than that of the existing method, it provided more repeatable material parameters, exhibiting better characteristic results.

Keywords: characteristic value · evaluation method · fibre reinforced concrete · material parameters · mixing model · three-point bending beam test

1 Introduction

Fibre reinforced concrete (FRC) is a composite material, where the concrete matrix is reinforced with fibres of various shapes such as straight, hooked, or wavy and various materials such as steel, polypropylene, or glass [1]; the fibres are short compared to the concrete elements. The addition of fibres increases the fracture energy of concrete, which also leads to higher ductility [2]. When testing and designing FRC, the location of the fibres in the concrete should not be ignored; currently, there are only a few recommendations for this.

A three-point (e.g. EN 14651) or four-point bending beam test (e.g. ASTM C1609/C1609M-19a) is generally used to determine the material parameters of fibre reinforced concretes. The results of the beam tests can be scattered owing to the relatively small reference area of the cross section of beams (150 × 150 mm) [3].

The number of fibres intersecting a cracked cross section is a key factor, as it influences the post cracking residual stress. Particularly, in a bent cross section, both the number and the location of the fibres is important, as fibres closer to the tension edge

of the beam can increase the residual moment capacity of the FRC cross section more effectively than those located close to the neutral axis. However, the number and location of the fibres are usually ignored during the evaluation of the test results. Traditional evaluation methods, such as the Student's t test, are unsuitable for engineering design since they yield low and even negative characteristic values of residual stress in certain cases, owing to their high standard deviations. Proceeding with the design process using material parameters derived from low characteristic values is uneconomical and produces a highly negative environmental impact and significant carbon footprint. These limitations can be overcome by improving the evaluation of material parameters while still ensuring safety.

Regarding evaluations, both the number and location of fibres have not yet been considered in any guidelines or standards despite the effect of these for such a small reference area is significant. Although many examples of the correlation between the number of intersecting fibres on a cracked cross section and the residual strength of FRC can be found in existing literature, no evaluation method that considers these factors has been proposed. This proposed evaluation method has already been proven useful in some of my former works [4].

2 Three-Point Bending Test

The number and location of fibres on a fracture surface are important details in evaluating the results of a beam test. The three-point notched bending beam test is the most suitable for this evaluation because both the crack initiation and fracture surface formation are controlled by the notch. The available standard for the three-point notched bending beam test is EN 14651, which is based on RILEM TC 162 [5]. In this study, the size of the beam was $150 \times 150 \times 550$ mm, the distance between the supports was $L = 500$ mm, and a concentrated load was applied to the middle of the beam. The speed of the loading device was CMOD controlled. The notch was 25 mm deep; the dimensions of the middle cross section were $b \times h_{sp} = 150 \times 125$ mm (Fig. 1a). During the loading process the CMOD is measured. The results of the test, including the load-CMOD diagram, are shown in Fig. 1b.

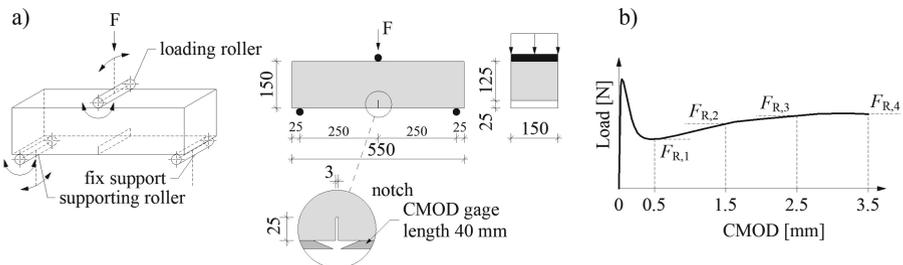


Fig. 1. a) Experimental setup: three-point bending test according to the European standard (EN), b) typical load-CMOD diagram of FRC.

During the evaluation, the residual stresses ($f_{R,1}, f_{R,2}, f_{R,3}$, and $f_{R,4}$) were calculated at specific values of CMOD (0.5, 1.5, 2.5, and 3.5 mm) using the following equation:

$$f_{R,i} = \frac{3F_{R,i}L}{2bh_{sp}^2} \quad (1)$$

3 Test Evaluation

3.1 Statistical Evaluation

The test results were used to determine the mean, characteristics, and design values of the proposed design. The mean values for the tested specimens were calculated as follows:

$$f_{R,i,m} = \frac{\sum_{j=1}^n f_{R,i,j}}{n} \quad (2)$$

Statistical methods for determining the characteristic value can be found in many standards and guidelines, including ISO 2394, EN 1990:2011, and RILEM TC 162 [5]. These methods assume a normal distribution, and the characteristic value is calculated as follows:

$$f_{R,i,k} = f_{R,i,m}(1 - k_n V_x) = f_{R,i,m} - k_n s_x \quad (3)$$

where:

$f_{R,i,k}$ = characteristic value of the residual flexural tensile strength, MPa,

$f_{R,i,m}$ = mean value of the residual flexural tensile strength, MPa,

$V_x = s_x/f_{R,i,m}$, coefficient of variations,

s_x = standard deviation, MPa,

k_n is factor dependent on the number of specimens [6], as shown in Table 1. The V_x unknown should be used if the coefficient of variation V_x is not known from prior knowledge.

Table 1. Values of k_n for the 5% characteristic value, according to EN 1990:2011.

n	1	2	3	4	5	6	8	10	20
V_x known	2.31	2.01	1.89	1.83	1.80	1.77	1.74	1.72	1.68
V_x unknown	–	–	3.37	2.63	2.33	2.18	2.00	1.92	1.76

3.2 Proposed Evaluation

The number and location of the fibres, which are relevant parameters, are ignored when determining the performance. In the proposed method, these parameters are incorporated into the evaluation to overcome the problems that occur when using the standard evaluation method. Fundamentally, the residual tensile strength of FRC is determined using a bending test. In such tests, not only the number of fibres intersecting the cracked section but also their distance from the compressed zone of the cross section is relevant. Even with the same number of fibres intersecting the cracked cross section, the load-CMOD curves derived from the FRC bending beam tests can exhibit a significant difference if the location of the fibres is different, for instance, if one sample has more intersecting fibres closer to the compressed zone than the other sample. In particular, the cross section that has more fibres closer to the compressed zone has a smaller moment capacity than the sample that has more fibres farther from the compressed zone, as shown in Fig. 2.

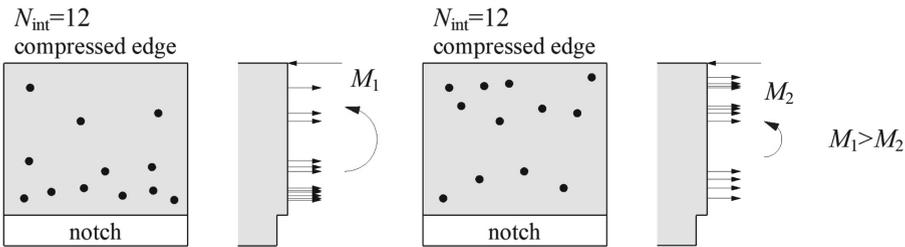


Fig. 2. Effect of the location of the fibres on a cracked cross section.

During the evaluation, the residual stresses ($f_{R,i}$) were determined using the results of three-point beam tests. In the statistical evaluation, the characteristic value of residual stress ($f_{R,i,k}$) was determined from the mean value ($f_{R,i,m}$) and standard deviation (s_x) obtained from the test results. In the proposed approach, the first step is determining the number and location of fibres on the surface intersecting the cracked cross section. Subsequently, given the geometry (length) and dosage of the fibres (kg/m^3 and N per unit volume), the number of fibres intersecting the cross section is determined using an analytical model assuming a uniform distribution. The characteristic number of fibres intersecting the cross section can be determined by considering the mean value and standard distribution of the analytical model. In the final step, the characteristic value of the residual stress is determined from the mean value of the test results and the results of the analyses of the cross sections and analytical model. This method can be used in cases where no fibre rupture occurs during the beam test until the maximum CMOD value is reached (in this case, until $\text{CMOD} = 3.5 \text{ mm}$).

Fibre-Moment. Generally, the residual force in fibres can be modelled as a predominantly permanent residual tension stress on the tension side of the section, as recommended by most guidelines [7–9]. This tension stress depends on the number of fibres intersecting the cross section. The fibre-moment (S_f) can be defined for the cross section in terms of the number and location of intersecting fibres. After conducting the beam test, the beam was severed into two parts, and the cracked cross section

was divided into five strips parallel to the notch, with a thickness of 2.5 cm (Fig. 3a). The number of fibres in each strip was determined. The compressed concrete zone was relatively small and negligible compared to the size of the cross section. Thus, the depth to the neutral axis was considered zero, and the depth to each strip was measured from the centre of each strip. The fibre-moment is calculated as follows:

$$S_f = \sum_{s=1}^5 t_s N_{\text{int},s} \quad (4)$$

where:

S_f = the fibre-moment, m,

t_s = the distance of the strip from the edge of the beam, as shown in Fig. 3a, m,

$N_{\text{int},s}$ = number of intersecting fibres in the actual strip.

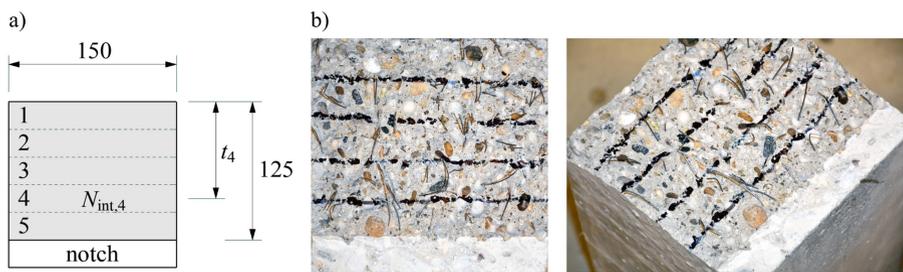


Fig. 3. Analysis of the cracked cross section: a) strips of the cross section, and b) image of analysed cross section.

Analytical Model for Fibre Mixing in Concrete. The variation in the fibre-moment at certain dosages was significant owing to the small number of intersecting fibres and the small reference area. The number of fibres intersecting the cracked cross section assuming ideal mixing, uniform distribution, and a known number of fibres per unit volume (N) can be determined using the analytical model proposed by Naaman [10]. This model provides the standard deviation and lower quantiles (5% fractile) of the number of intersecting fibres [4, 6]. The mean value of the probability of a fibre intersecting a crack in the case of N fibres in volume V is:

$$N_{m,a} = 0.5N \quad (5)$$

where:

$N_{m,a}$ = the number of fibres intersecting the reference area of the beam,

N = number of fibres in volume V ,

V = volume of a brick with a dimension of $150 \times 125 \times l_f$, mm^3 ,

l_f = length of the fibre, mm.

The standard deviation of the number of fibre intersecting is:

$$\sigma = \sqrt{0.25N} \quad (6)$$

The number of fibres intersecting the cross section at a 95% confidence level (characteristic value) is:

$$N_{k,a} = N_{m,a} - 1.6449\sigma \quad (7)$$

Analytical Fibre-Moment. Using the analytical model, the fibre-moment can be determined using the mean and characteristic values of the number of fibres intersecting the cross section. In the analytical model, the fibres are assumed to have a uniform distribution on the cracked cross section. Therefore, the distance from the centre to the compressed edge can be considered the moment arm. The mean and characteristic values of the analytical fibre-moments ($S_{f,m,a}$ and $S_{f,k,a}$) are:

$$S_{f,m,a} = N_{m,a}62.5 \text{ mm} \quad (8)$$

$$S_{f,k,a} = N_{k,a}62.5 \text{ mm} \quad (9)$$

Evaluation Method. The fibre-moments of the test specimens ($S_{f,j}$) and the corresponding residual strengths ($f_{R,i,j}$) calculated using Eq. (1) are approximately directly proportional, that is, a larger fibre-moment corresponds to a larger residual strength. Consequently, a linear function, known as a reference function, can be fitted to the data of the complete set using linear regression.

Cavalaro and Aguado [11] classified different types of scattering into three types: a) testing procedure scatter, b) production process scatter, and c) intrinsic scatter. Intrinsic scatter, which accounts for the maximum proportion of the scatter in the results, can be attributed to the random locations of the fibres. In the proposed approach, the testing procedure and production process scatter are considered using the characteristic reference function, and the intrinsic scatter is considered in terms of the characteristic value of the analytical fibre-moment, as shown in Fig. 4a and 4b. The distribution of the signed differences between the residual strengths of the test results ($f_{R,i,j}$) and the value of the reference function at the corresponding fibre-moment ($S_{f,j}$) can be estimated by considering a normal distribution. In this case, the lower quantiles can be estimated as follows: the $\Delta f_{R,i,j}$ value for every test result is determined as the distance from the reference line according to Fig. 4b. The corrected sample standard deviation is calculated as follows:

$$s_{\Delta} = \sqrt{\frac{\sum_{j=1}^n \Delta f_{R,i,j}^2}{n-1}} \quad (10)$$

The offset of the characteristic reference function (Fig. 4b), c , can be calculated as follows:

$$c = -k_n s_{\Delta} \quad (11)$$

Using the reference function, characteristic reference function, and mean and characteristic fibre-moments, the mean ($f_{R,i,m}$) and characteristic ($f_{R,i,k}$) values of the residual strength can be calculated, as shown in Fig. 4b.

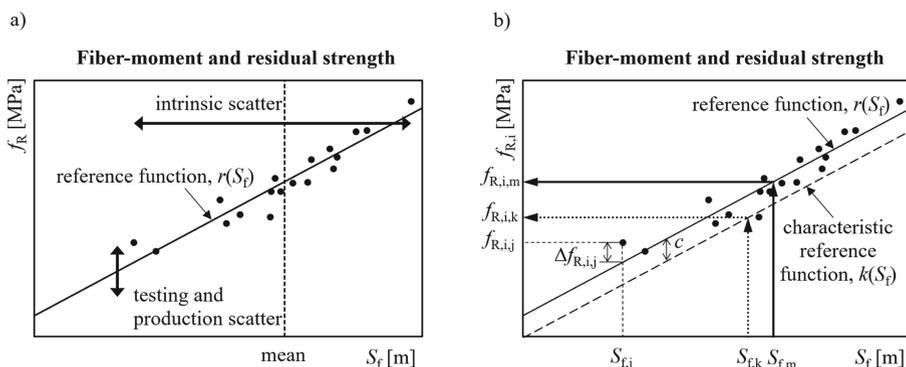


Fig. 4. a) Intrinsic, testing procedure, and production process scatter, and b) calculation of the mean and characteristic value of the residual strength by using the characteristic reference function.

4 Laboratory Tests

The FRC beams were fabricated and tested using steel and synthetic fibres according to the EN 14651 standard. The mix design is listed in Table 2. The geometries and types of fibres are listed in Table 3. Six specimens of each type were prepared. The experimental results for different dosages of steel and synthetic fibres are shown in Fig. 5.

Table 2. Concrete mix design.

Component	Quantity (kg/m ³)
Coarse aggregate (8–16 mm)	428
Aggregate (4–8)	744
Sand (0–4 mm)	558
Limestone powder	130
Cement (CEM I 42.5 R)	350
Water	192.5
Superplasticizer	1–4

Table 3. Material properties of the fibres.

Property	Steel fibre (ST)	Synthetic fibre (SY)
Tensile strength, MPa	700	640
Elastic modulus, GPa	200	12
Diameter/length, mm	1/50	0.7/48
Fibres/kg	3,350	59,520

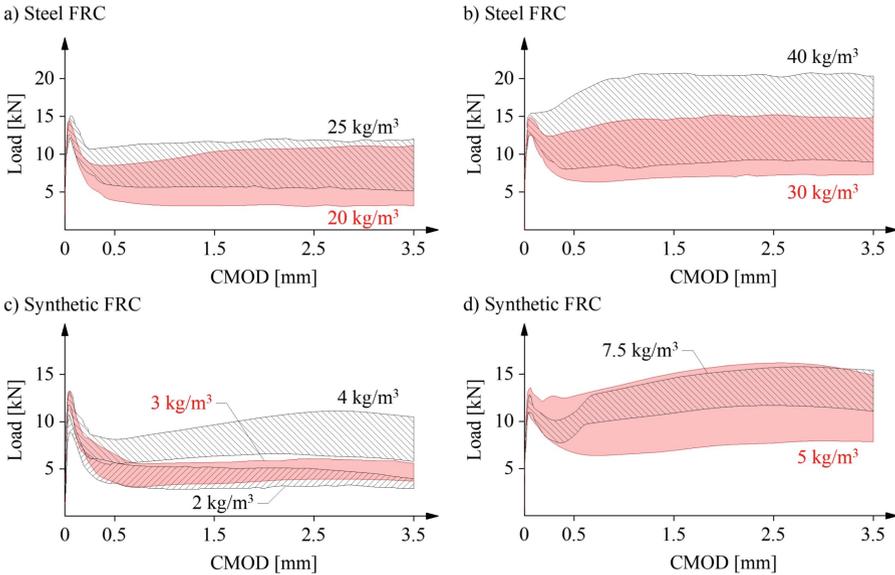


Fig. 5. Envelope load-CMOD diagrams of a) and b) steel, c) and d) synthetic FRC.

5 Results

5.1 Results of Analytical Mixing Model

The accuracy of the analytical model could be examined by comparing the number of fibres intersecting the cross section with the results of the analytical model, as shown in Fig. 6.

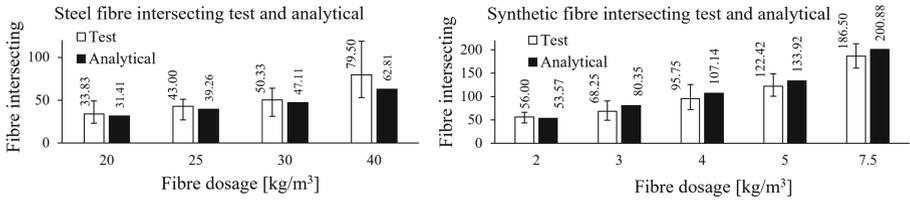


Fig. 6. Comparison of the analytical prediction and test results for the number of fibres intersecting the cracked cross section, a) steel fibre and b) synthetic fibre.

5.2 Correlation of the Fibre-Moment and Residual Strengths

The given fibre-moments and the corresponding residual strengths under the laboratory tests ($f_{R,1}$ and $f_{R,4}$ for a CMOD of 0.5 and 3.5 mm, respectively) for both steel and synthetic fibres are illustrated in Fig. 7.

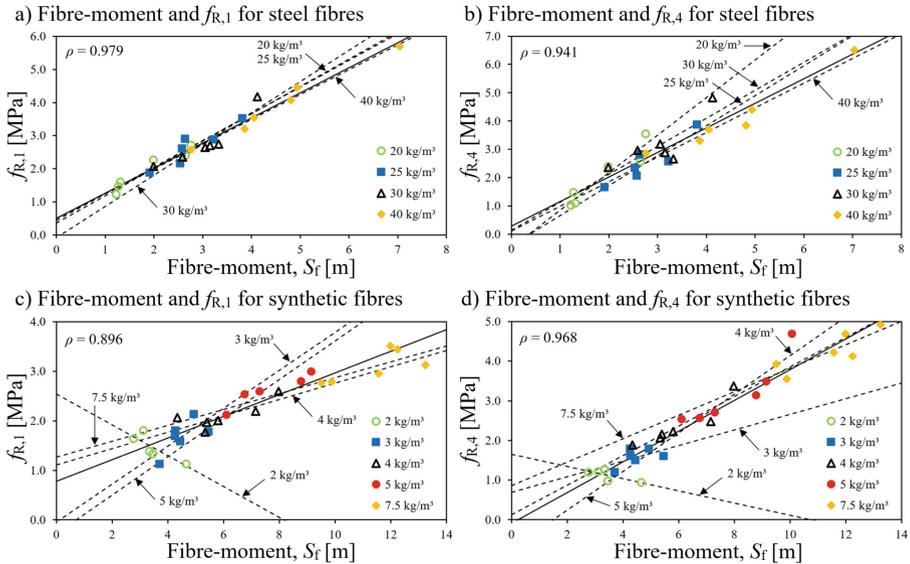


Fig. 7. Fibre-moment and residual strengths, along with the reference function a) $f_{R,1}$ for steel fibre, b) $f_{R,4}$ for steel fibre, c) $f_{R,1}$ for synthetic fibre, and d) $f_{R,4}$ for synthetic fibre.

5.3 Comparison of Characteristic Values Under Statistical Analysis and Proposed Method

The mean and characteristic values of the residual strength under both the proposed and statistical analysis methods were compared, as shown in Table 4.

Table 4. Mean and characteristic values of the residual strength under the statistical and proposed evaluation methods.

Mean and characteristic values of residual strengths for steel FRC, MPa								
dosage kg/m ³	Statistical evaluation				Proposed evaluation			
	$f_{R,1,m}$	$f_{R,1,k}$	$f_{R,4,m}$	$f_{R,4,k}$	$f_{R,1,m}$	$f_{R,1,k}$	$f_{R,4,m}$	$f_{R,4,k}$
20	1.938	0.645	2.010	-0.15	1.938	1.271	2.010	0.793
25	2.658	1.380	2.556	0.897	2.386	1.513	2.207	1.034
30	2.778	1.188	3.146	1.258	2.690	1.634	3.054	1.485
40	3.925	1.555	4.103	1.304	3.437	2.685	3.536	2.398

Mean and characteristic values of residual strengths for steel FRC, MPa								
dosage kg/m ³	Statistical evaluation				Proposed evaluation			
	$f_{R,1,m}$	$f_{R,1,k}$	$f_{R,4,m}$	$f_{R,4,k}$	$f_{R,1,m}$	$f_{R,1,k}$	$f_{R,4,m}$	$f_{R,4,k}$
2	1.455	0.870	1.115	0.791	1.495	1.334	1.136	0.999
3	1.686	0.964	1.583	1.102	1.686	0.949	1.583	1.064
4	2.100	1.492	2.364	1.202	2.100	1.613	2.364	1.678
5	2.847	1.399	3.183	1.383	2.847	1.902	3.182	2.038
7.5	3.097	2.394	4.238	3.143	3.097	2.457	4.239	3.387

6 Conclusions

The advantages of the proposed evaluation method over existing statistical methods are clearly visible. Regarding synthetic fibres at a dosage of 2 kg/m³, the slope of the dosage reference function was negative, leading to an uninterpretable evaluation. However, this evaluation could be performed using the total data. According to the statistical approach, at a dosage of 20 kg/m³ of steel fibres, $f_{R,4,k}$ was negative; however, no irregularity occurred at the corresponding $f_{R,1}$ values. In almost all the cases, the characteristic values were higher than those under the statistical method, and positive characteristic results were obtained. Noteworthy, the mean and characteristic values increased in all the cases when the fibre dosage was increased, unlike the values under the statistical methods. The fibres were aligned because the vibration could be accounted for by considering the fibre-moment, and the overestimated mean parameter could be eliminated.

The most notable limitation of the proposed approach is the additional labour required to determine the number and location of fibres in the cracked cross section. The proposed method is useful when a residual strength pertaining to different dosages needs to be determined.

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