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Analytical model for rigid (steel) and flexible (synthetic) fibre mixing in concrete

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Abstract. Quantity of the fibres intersecting the unit cross section is a relevant parameter. Firstly, the residual tension strength, which is the base of the material model of the fibre reinforced concrete, depends most on the number of fibres intersecting the cracked cross section. Secondly, by counting the fibres on the cross section of the broken test specimen the non-uniform distribution of the fibres can be examined and taken into account during the evaluation. Therefore, for most of the material model the initial parameter is the quantity of fibres intersecting the unit cross section.

1. Introduction

Romualdi and Mandel already worked on determining this quantity at the early scientific research of fibre reinforced concrete [1], where the behaviour of the fibre reinforced concrete and the quantity of fibres intersecting the cracked cross section of the specimen are analysed through experiments. In 1972 Naaman [2] determined the quantity of fibres intersecting the cross section by supposing uniform distribution and applying probability analysis in his doctoral thesis. Krenchel, who provided a calculation method based on empirical results [3], introduced the orientation factor for describing the orientation of the fibres, which became the base for fibre reinforced material models. Among others, Soroushian and Lee [4], Ng, Foster and Htut [5], Lee, Cho and Vecchio [6] and Dupont and Vandewalle [7] were all engaged in determining this orientation factor.

Orientation of the fibres changes near the formworks, and by that the orientation factor is altered as well, which affects the quantity of the fibres intersecting the unit cross section. The literature calls this phenomenon wall-effect. In literature, beams with square cross section are usually examined, in which case the wall-effect is present at the sides (one side of the mould) and at the corners (two sides of the mould). Dupont and Vandewalle [7] and Stroven [8] investigated how wall-effect influenced the orientation factor in the case of steel fibres. Zerbino et al. [9] examined the location of steel fibres in self compacting concrete elements, in such elements as slab, wall and beam, which was similar to Sarmiento et al [10], who compared the experimental results with the numerical ones in the case of steel fibres located in the beams.

The wall-effect is different in the case of synthetic fibres: while steel fibres rotate when coming in contact with the formwork, then the synthetic fibres react by bending. Oh, Kim and Choi [11] did not take this effect into consideration when analysing synthetic fibre reinforced beams, however Alberti et al. [12] suggested to consider flexibility of the fibres in modelling.

In this paper various methods will be demonstrated that help determining the quantity of the fibres intersecting the cross section. The wall-effect and the variation of the orientation factor will be



examined. The rigid (steel) and the flexible (synthetic) fibres will be differentiated. Furthermore, calculation methods of the orientation factor will be presented, by which the quantity of fibres intersecting the cross section can be determined. Predictions of the analytical model will be compared to experimental results.

2. Determining the quantity of fibres intersecting the unit cross section

Quantity of the fibres intersecting the unit cross section is the basis for most of the material models that consider fibres discretely. Orientation of the fibres is also a relevant parameter for evaluating the experiments, due to which on one hand, the quality of the mixing can be concluded [7], while on the other hand the effect of non-uniform location of the fibres on the cracked cross section can also be considered when evaluating the test results [13]. Numerous researchers examined the impact of mixing on the experimental results regarding different types of elements [9, 10].

2.1. Derivation by Romualdi and Mandel (1964)

Fibres parallel or nearly parallel to the tensile stress are effective in crack control. Thus, some correction must be made for those fibres that are ineffectively oriented. Romualdi and Mandel [1] assumed that the ratio of the average of the projected lengths in one direction to the total length is a proper correction. Based on this the average length of fibres with a given midpoint and length of l_f projected to axis x with uniform distribution of α and γ is given by:

$$l_{f,x,m,1} = \frac{\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} l_f \cos \alpha \cos \gamma d\alpha d\gamma}{\left(\frac{\pi}{2}\right)^2} = 0.405l_f \quad (1)$$

Romualdi and Mandel derived the quantity of fibres intersecting the unit cross section from the average distance of midpoint of the fibres. This is given by the following:

$$n = \frac{l_{f,x,m,1}N}{V} = 0.405l_f \frac{N}{V} \quad (2)$$

where n is the quantity of the fibres intersecting the unit cross section [pc/m^2], and N is the quantity of the fibres in the volume V .

If both α and γ are distributed uniformly, the endpoints on the sphere densify around the axis z (figure 1b). This is unrealistic. It is well-known, that uniform distribution on the surface of the sphere can be achieved by uniform distributions of the azimuthal angle and the height of the endpoint (figure 1d).

In this case average length of the fibres in the projection of direction x with uniform distribution of α and Z is given by:

$$l_{f,x,m,2} = \frac{\int_0^{\frac{\pi}{2}} \int_0^{\frac{l_f}{2}} l_f \cos \alpha \cos \left(\arcsin \frac{Z}{l_f} \right) dZ d\alpha}{\frac{\pi}{2} l_f} = 0.5l_f \quad (3)$$

The formula of Romualdi and Mandel, equation (3) has to be modified for the distribution of the end point of the fibres on the sphere to be uniform, thus the quantity of the fibres intersecting the unit cross section are modified as given below:

$$n = \frac{N}{V} 0.5l_f \quad (4)$$

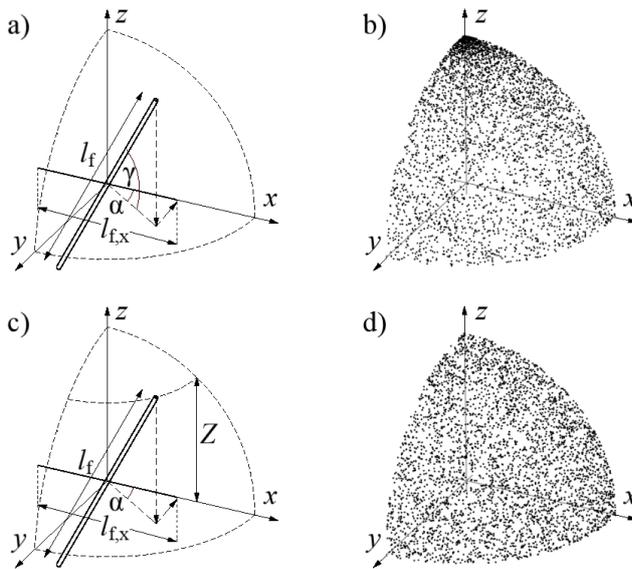


Figure 1. a) Determining spatial distribution of the fibres by polar coordinates α and γ b) Endpoints of the fibres with uniform distribution of α and γ c) Determining spatial distribution of the fibres by parameters α and Z d) Endpoint of fibres with uniform distribution of α and Z .

2.2. Derivation by Naaman (1972)

Naaman [2] determined the quantity of fibres intersecting the cross section by probability analysis. A fibre with a midpoint on axis x is to be considered here (figure 2).

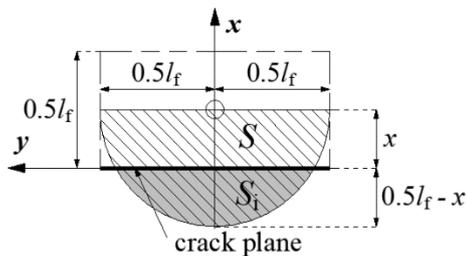


Figure 2. Probability of the fibre intersecting the crack plane.

If the midpoint of the fibre is located at a distance smaller than $0.5l_f$, then the crack plane can be intersected by the fibre. The probability of this is the quotient of surface of spherical cap S_i and half-sphere S drawn around the midpoint of fibre according to figure 2.

$$q_{Naaman} = \frac{S_i}{S} = \frac{0.5l_f - x}{0.5l_f} = 1 - \frac{2x}{l_f} \tag{5}$$

Considering unit volume at both sides of the crack plane, the quantity of fibres intersecting the unit cross section is similar to the obtained formula by uniform distribution on the sphere (equation (4)):

$$n = 2 \int_0^{\frac{l_f}{2}} \left(1 - \frac{2x}{l_f}\right) \frac{N}{V} dx = \frac{N}{V} 2 \int_0^{\frac{l_f}{2}} \left(1 - \frac{2x}{l_f}\right) dx = \frac{N}{V} 0.5l_f \tag{6}$$

2.3. Orientation factor of Krenchel (1975)

Supposing an ideal state each fibre is perpendicular to the crack plane, therefore in the current case each fibre is facing direction x . Considering the volume fraction of the fibres: V_f (weight of the fibres in concrete [kg/m³]/bulk density of fibres [kg/m³]), and the cross section area of a discrete fibre: A_f . In this case the quantity of the fibres intersecting cross section A is given by the following formula:

$$n_i = \frac{V_f}{A_f} A \tag{7}$$

where n_i indicates the ideal quantity [pc] in cross section A .

As orientation of the fibres is not ideal in reality Krenchel [3] introduced the orientation factor with the following formula:

$$\theta_{\text{Krenchel}} = \frac{n_a}{n_i} = n_a \frac{A_f}{V_f A} \tag{8}$$

where n_a is the quantity [pc] of fibres intersecting cross section A , determined by experiment by counting at the cracked surface or sliced section.

It is therefore apparent that orientation factor of Krenchel is no other than the factor modifying fibre length in the derivation by Romualdi and Mandel and Naaman, which, supposing uniform distribution, is 0.5.

3. Wall-effect

In the case of rigid (steel) and flexible (synthetic) fibres the effect of the formwork to the orientation of the fibres is different, as fibres of different materials behave distinctly when coming in contact with the formwork. This distinct behaviour will be presented with its effects to the quantity of fibres intersecting the cross section.

3.1. Wall-effect in the case of rigid (steel) fibres

3.1.1. Model of Dupont and Vandewalle for steel fibres

The researchers divided a square cross section of a beam to three regions considering the effect of formworks: 1: undisturbed zone, 2: disturbed zone – one side of the mould, 3: disturbed zone – two sides of the mould (corner) (figure 3).

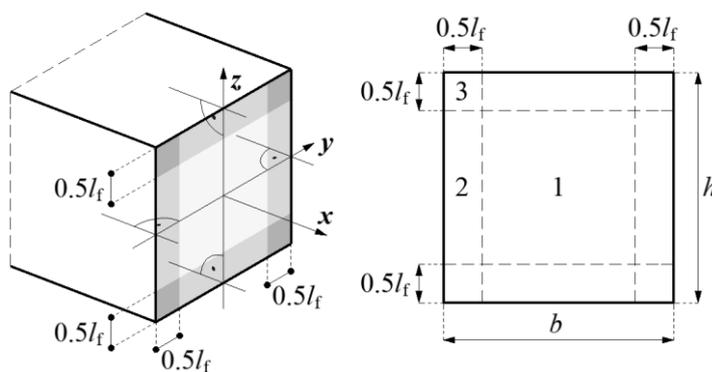


Figure 3. Cross section zones [7].

It was supposed that the midpoint of the fibres near the formwork does not change, and thus their distribution stays uniform, however those fibres that meet the formwork swift around their midpoint in such a way that they contact the formwork. This is true for those fibres having their midpoint closer to the formwork than a distance of $0.5l_f$. These factors were considered with various orientation factors and then the quantity of fibres intersecting the cross section was determined according to formula of Krenchel, equation (7).

The formula of Romualdi and Mandel equation (4) and Krenchel equation (7) start with the condition that the direction of the fibres is vertical at the cross section, then a modified fibre length is used for calculation instead of the actual one. The former is using the effective fibre length and the latter is applying the fibre length modified by the orientation factor. Both of the researchers had similar results. Based on this model only those fibres are influenced by the wall-effect which are able to come in contact with the formwork, thus the midpoint of the fibre is closer to the formwork than a distance of $0.5l_f$.

3.1.2. Suggested model

Coming back again to the derivation of Naaman, it should be examined what happens if the fibres are analysed in an infinite rectangular cylinder. Let us consider a space named T_1 that is $0.5l_f$ wide, $0.5l_f$

deep and has infinite length in direction z , in which the probability of its fibres intersecting the face A_1 is examined according to figure 4. Midpoints of the fibres are located in this space T_1 , the origin is located at the edge of the face A_1 . The probability of intersecting the face A_1 by fibres with midpoint coordinates of x and y is given by:

$$\theta(x, y) = \frac{S_i(x) - (S_{n1}(x, y) + S_{n2}(x, y))}{S} \tag{9}$$

where $S_{n,1}$ and $S_{n,2}$ is a surface that belongs to those fibres which are intersecting the plane of the A_1 , but not the A_1 according to figure 4a. Let us consider two adjacent spaces with a size similar to T_1 , named T_{2a} and T_{2b} . Fibres with a midpoint of $(x; 0.5l_f + y)$ and $(x; -y)$ located here and the surfaces belonging to fibres with these midpoints, which extend to space T_1 will similarly be S_{n1} and S_{n2} as can be seen in figure 4b.

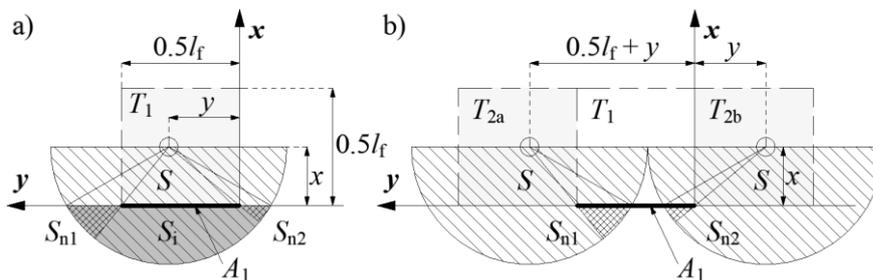


Figure 4.
a) interpretation of fibres intersecting the face A_1 from space T_1
b) penetration from the adjacent spaces T_{2a} and T_{2b} .

In the case of fibres located in space T_1 with coordinate y the probability of intersecting the face of A_1 is given by:

$$\theta(y) = 2 \int_0^{0.5l_f} \frac{S_i(x) - (S_{n1}(x, y) + S_{n2}(x, y))}{S} dx \tag{10}$$

and in the case of fibres located at space T_{2a} and T_{2b} with coordinate y the probability of intersecting is:

$$\theta(y) = 2 \int_0^{0.5l_f} \frac{S_{n,i}(x, y)}{S} dx \tag{11}$$

The probability of certain fibres with midpoint coordinates of y intersecting the face A_1 is demonstrated in figure 5.

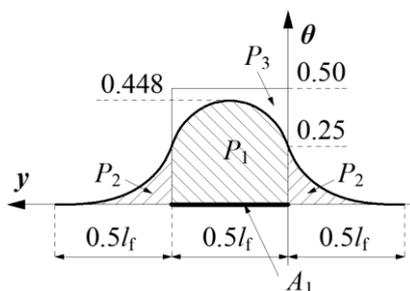


Figure 5. The probability of intersecting of the fibres.

Moving away from the centre of face A_1 until the edge in space T_1 the probability is decreasing as the sum of $S_{n1}+S_{n2}$ is increasing, respectively, moving away from the edge of face A_1 the probability is also decreasing in space T_2 . The area below the curve of face A_1 is named P_1 and the area above the curve completing up to 0.50 is named P_3 . However the face A_1 can also be intersected by fibres from adjacent spaces T_{2a} and T_{2b} . The corresponding area in figure 5 will be named P_2 . Carrying out the integration the result is $2P_2=P_3$, so by considering intersecting of fibres reaching over from adjacent spaces T_{2a} and T_{2b} , the outcome is given below:

$$\theta_{T_1:T_{2a}-T_{2b}} = \frac{P_1 + 2P_2}{0.5l_f} = \frac{0.196947 + 2 \cdot 0.026527}{0.5} = 0.50 \tag{12}$$

which matches the orientation factor of Naaman derived to infinite space: equation (6).

It should be examined what happens if there is a mould next to the adjacent space T_1 , at the border of the similar sized space T_3 , as it can be seen in figure 6a. Fibres located at this space can contact the formwork, so the formwork can impact their orientation, fibres rotate. Due to the changing of the orientation reaching over of the fibres from space T_3 to T_1 is limited, therefore fewer fibres are able to intersect face A_1 . Space T_3 can be divided into three clearly separable parts, interpreted in the plane of xy . If midpoint of the fibre is in field I, then it does not intersect face A_1 , if it is in field II, then it does, but due to the wall-effect there are fewer fibres than without the mould. If midpoint of the fibre is in field III they are not affected by the wall-effect at all, but can intersect the face A_1 . The explanation and demonstration of these fields can be seen in figure 6c-f.

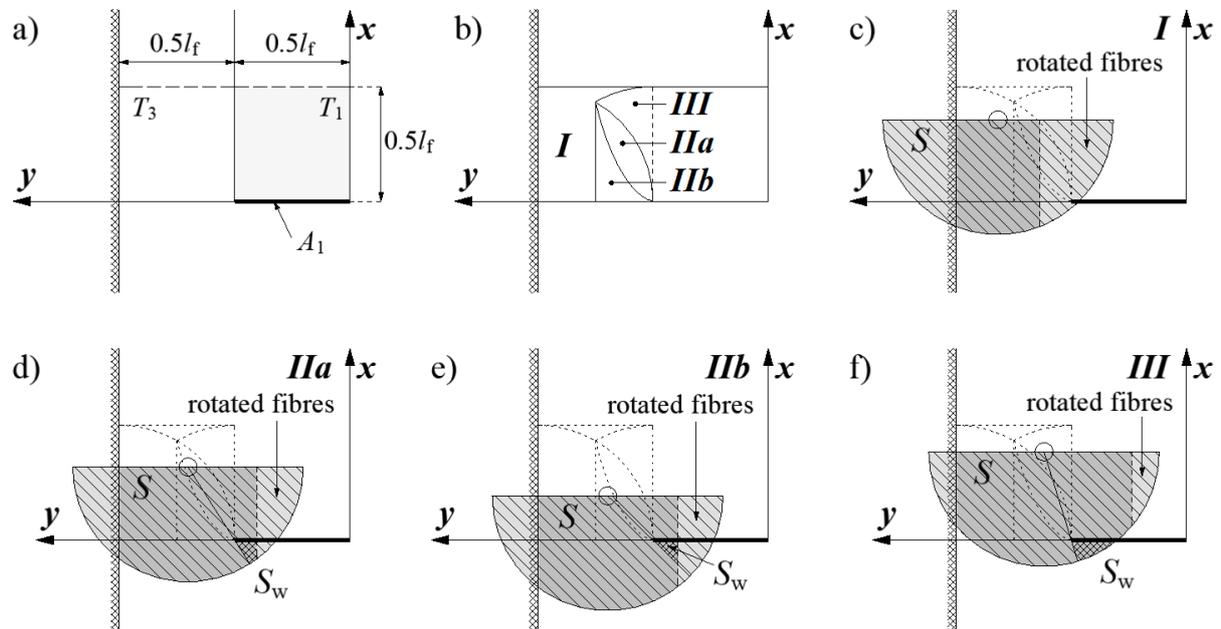


Figure 6. a) T_1 and T_3 , mould at border of T_3 b) Defining fields I, IIa, IIb and III c) Explanation of fields I d) IIa e) IIb f) III.

In this case the probability of intersecting face A_1 :

$$\theta(y) = 2 \int_{II-III} \frac{S_w(x,y)}{S} dx \tag{13}$$

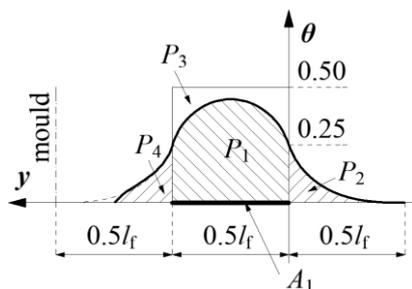


Figure 7. The probability of intersecting face A_1 if the space of A_1 is bounded by a continuous space from one side and by a space concerned by wall effect from the other side.

Probability depicted at axis y is in figure 7. The probability of fibres in space T_1 intersecting face A_1 considering the adjacent and wall-effect free space T_2 and also space T_3 that is affected by the wall-effect from the other side can be explained as the following:

$$\theta_{T_1:T_3-T_2} = \frac{P_1 + P_2 + P_4}{0.5l_f} = \frac{0.196947 + 0.026527 + 0.025418}{0.5} = 0.497 \tag{14}$$

Fibres in space T_1 are not able to contact the mould, therefore the formwork does not have an impact on their orientation, only on the fibres in space T_3 . However due to the limited over-reaching of the fibres still fewer fibres intersect face A_1 in this case, thus we are talking about a *quasi-wall-effect* here. It should be examined what happens in space T_3 where the formwork affects orientation of the fibres directly (figure 8). Fibres in this space can contact the formwork and their orientation changes. In this case probability of the intersecting is:

$$\theta(y) = 2 \int_0^{0.5l_f} \frac{S_i(x)}{S(x, y)} dx \tag{15}$$

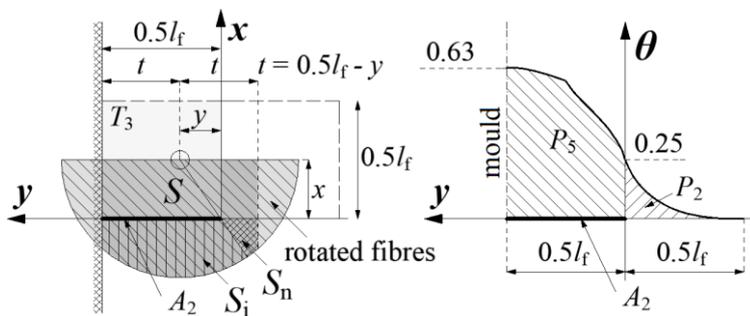


Figure 8. a) Fibres in space T_3 intersecting the face A_2
 b) The probability of intersecting face A_2 if space T_3 is bounded by a continuous space from one side and a wall from the other side.

The probability of fibres in space T_3 intersecting face A_2 considering the fibres reaching over from the adjacent space T_1 is given below:

$$\theta_{T_3:zs-T_1} = \frac{P_5 + P_2}{0.5l_f} = \frac{0.262373 + 0.026527}{0.5} = 0.578 \tag{16}$$

The orientation factor of the fibres next to the formwork is $\theta=0.63$, which matches with the orientation factor of fibres located in the plane $\theta=2/\pi=0.63$ [4, 14].

3.2. Wall-effect in the case of flexible (synthetic) fibres

It is supposed that synthetic fibres bend when coming in contact with the formwork. Alberti et al [12] considered this by decreasing the length of fibres, in current dissertation it is considered by the bending of the fibres. The effect of the formwork in this case is only present at those fibres which have their midpoint closer to the formwork than the distance of $0.5l_f$. In the case of synthetic fibres the reaching over is not limited due to the bending of the fibres, thus the *quasi-wall-effect* does not exist (figure 9).

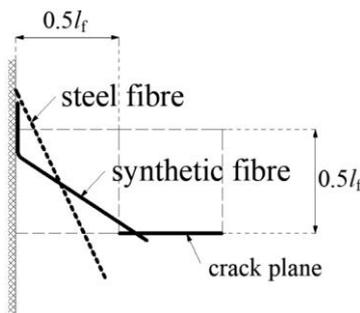


Figure 9. Over-reaching of steel and synthetic fibres to the adjacent cross section zone when the wall-effect is active.

As endpoints of the fibres will not be on the surface of sphere after the bending, the probability analysis method of Naaman cannot be used in this case. At the same time its utilization is not required, as the wall-effect does not have an impact on the adjacent space. Thus for determining the quantity of fibres intersecting the cross section in the case of synthetic fibres the modified formula of Romualdi and Mandel derived from the average fibre length equation (4) is more suitable. For this the average length of projection in direction x of a fibre needs to be determined. Fibres contacting the formwork bend, their projection length of direction x is considered by modifying equation (3):

$$l_{f,x,m,3} = \frac{\int_0^{\frac{\pi}{2}} \int_0^W \int_0^{\frac{\pi}{2}} l_f \cos \alpha \cos \left(\arcsin \frac{Z}{l_f} \right) dZ d\alpha + \int_0^{\frac{\pi}{2}} \int_0^{\frac{2}{W}} \left[l_f \frac{W}{Z} \left(\cos \left(\arcsin \frac{Z}{l_f} \right) - 1 \right) + l_f \right] \cos \alpha dZ d\alpha}{\frac{\pi}{2} l_f} \quad (17)$$

where W is the distance of the fibres from the formwork.

In the case of synthetic fibres the wall-effect is not symmetrical as it is in the case of steel fibres (figure 10), so the average of the projection length in direction x will be the following:

$$l_{f,x,m,4} = \frac{l_{f,x,m,2} + l_{f,x,m,3}}{2} \quad (18)$$

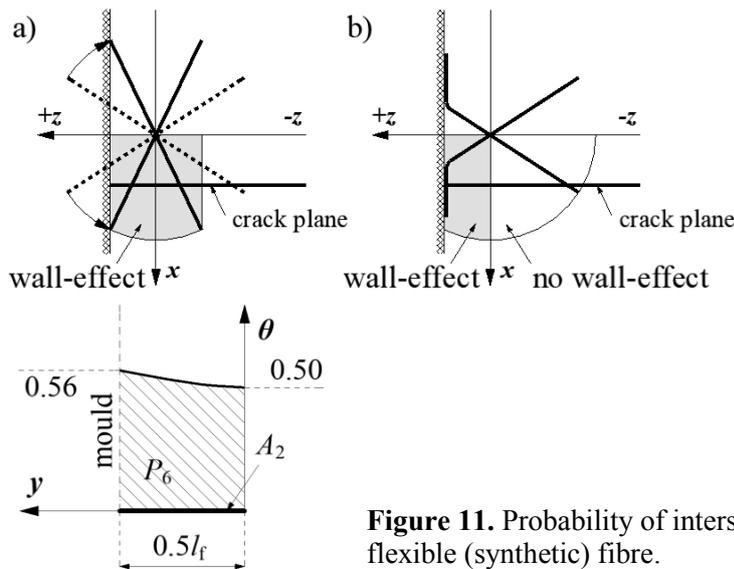


Figure 10. The impact of wall-effect on the quantity of fibres intersecting the cross section in the case of a) steel fibres and b) synthetic fibres.

Figure 11. Probability of intersecting of face A_2 in the case of flexible (synthetic) fibre.

Probability is demonstrated in figure 11. Thus the probability of fibres in space T_3 intersecting face A_2 in the case of flexible (synthetic) fibres is:

$$\theta_{T_3:zs} = \frac{P_6}{0.5l_f} = 0.53 \quad (19)$$

4. Orientation factors of rigid (steel) and flexible (synthetic) fibres

Steel fibre zones of Dupont and Vandewalle [7] can be seen in figure 12a, modified zones based on current study can be seen in figure 12b, and synthetic fibre zones can be seen in figure 12c, accompanied by their respective orientation factors in each case.

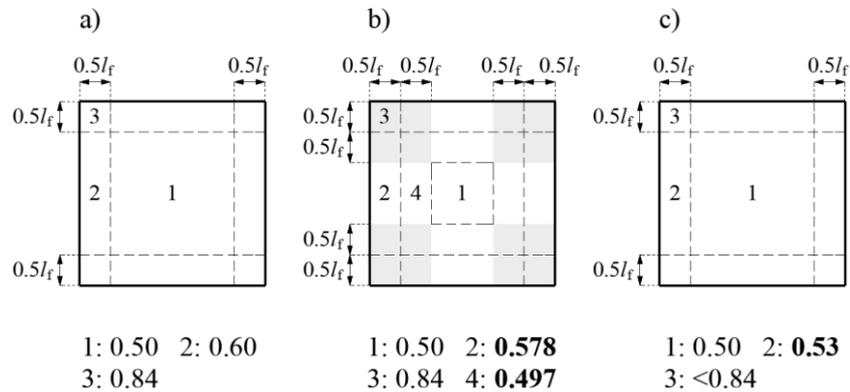


Figure 12. Cross section zones and their orientation factors.

5. Verification

Four steel and four synthetic fibre reinforced concrete beams were made. The pouring method was different: two of the beams were poured parallel to the longitudinal axis of the beam, and two of the beams were poured perpendicular. With this the effect of the different pouring method was taken into account. Cross section size of the beams was 150x150 mm, their lengths was 1 meter. The determination of the fibre dosage was carried out in order to have an equal number of fibres in each of the beams. For steel fibres there were 3181 fibres in 1 kg, while for synthetic fibres there were 35714 fibres. Thus the chosen dosage for steel fibres was 33.73 kg/m³, while for synthetic fibres it was 3 kg/m³, so there were 107346 fibres in 1 m³ of concrete.

The complete beams were cut to slices of 50 mm in longitudinal direction, then the fibres were counted in zones 1, 2, 3 and 4 in the case of steel fibres, and in zones 1, 2 and 3 in the case of synthetic fibres. Histograms, standard normal probability density function, test and model mean value of the fibres intersecting the cross sections can be seen in figure 13.

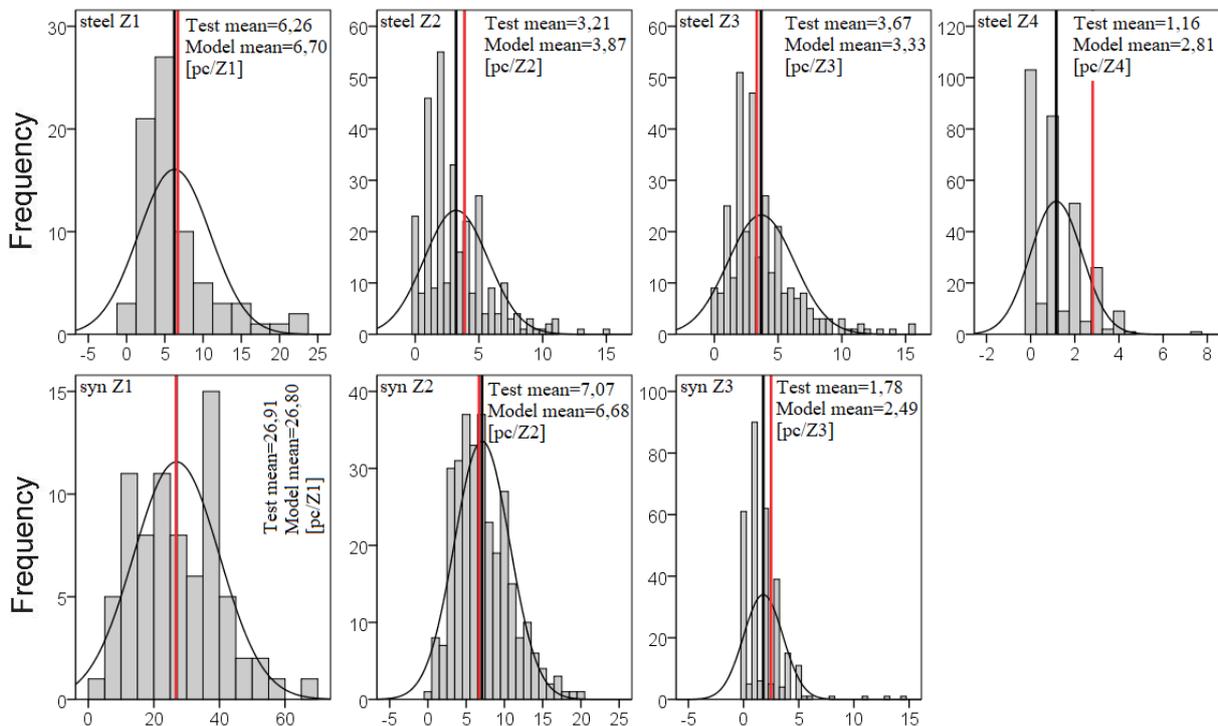


Figure 13. Histograms of steel and synthetic fibers intersecting the cross sections.

6. Summary

The major difference between the steel and synthetic fibres regarding their behaviour in concrete is the rigidity of discrete fibres: while steel fibres can be regarded as rigid, then synthetic fibres can be flexible. Based on current model fibres are described by their midpoint and orientation by supposing uniform distribution, from which the quantity of fibres intersecting the cracked surface can be derived. Orientation changes near the formwork, it becomes partly directional. This directionality is different for steel fibres as it is for synthetic ones, therefore the quantity of fibres intersecting the cross section also changes in a different way near the formwork.

In this paper a suggested model to steel fibres was presented, introducing the *quasi-wall-effect*, indicating that the impact of the wall influences wider areas of the cross section. Quantity of fibres intersecting the cross section can be determined by the model. Wall-effect works in a different way in the case of the synthetic fibres than it does for steel fibres. It was demonstrated that there is no quasi-wall-effect in the case of the synthetic fibres, and due to their flexibility when coming in contact with the formwork the orientation factors also change. Orientation factors for synthetic fibres was provided as well. The analytical model was verified both for steel and synthetic fibres.

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