Analytical model verification for FRP and synthetic fibre reinforced concrete beams

Peter Schaul and Gyorgy L. Balázs

Department of Construction Materials and Technologies,
Budapest University of Technologie and Economics,
Műegyetem rkp. 3, 1111 Budapest, Hungary

Abstract

Synthetic reinforcements in concrete structures, such as macro synthetic fibres and fibre reinforced polymer (FRP) bars are becoming more widely used nowadays, because of their most important advantage to be resistant against electrolytic corrosion. The FRP bars can provide flexural capacity both as main reinforcement and shear capacity as stirrups. Macro synthetic fibres can increase the ductility of the elements and the shear capacity of the concrete structures as well. For design with FRP and synthetic fibre reinforcement concrete structures some guidelines but there is no standard available for them (except *fib* MC2010). In this article an analytical model was developed to predict the shear capacity of the synthetic material reinforced concrete beams. The development of the formula and the verification of this formula will be presented. The analytical model takes into consideration the elastic modulus of the FRP material and the additional shear capacity from synthetic fibre reinforced concrete. In this way a complex model can be constructed that include contributions of only polymeric materials for tension and for shear in addition to concrete.

1 Introduction

The shear and the punching shear failure is one of the most dangerous failure modes in all structural elements especially in reinforced concrete slabs and beams. The phenomena of the shear is complex, contains several different components [1]. During the shear failure of the reinforced concrete beams an inclined crack is appearing on the side of the concrete beam, and the beam start to separate along the crack. The mechanism can happened very quickly and it can cause the total failure of the beam. Because this failure mode is very brittle all the reinforced concrete standards (ACI, JSCE, Eurocode) have a different section for the shear design. These standards usually recommend to calculate the shear capacity of the concrete and the web reinforcement separately.

These standards specify the stirrups as they are made from steel and the formulas are only valid for steel, respectively. However, the synthetic reinforcements, such as fibre reinforced polymer (FRP) rebars and synthetic fibre reinforcement in concrete is becoming a well-used alternative non- metallic reinforcement for concrete structures. The FRP bars are made from longitudinal fibres, usually glass, carbon and basalt, and from a thermoset or a thermoplastic resin. The fibres bear the load and the resin protects the fibres and transfers the loads to the fibres. Usually these bars have an orthotropic behaviour because of the manufacture. The process of the manufacture called pultrusion. The bars can be used as main reinforcement and as stirrups also, but with using thermoset resin the bars cannot be bent after the manufacture procedure. With using thermoplastic resin the bars can be formed after the pultrusion as well with adding heat to the bar, however, the strength of the bar will be lower. Important fields for using FRP bars are the MRI rooms in hospitals, tramlines, where no magnetic material can be used. Also an alternative reinforcement can be the FRP bars in concrete roads and bridges where the electrolytic corrosion can be significant.

Synthetic macro polymer fibres became a rather well-used material in concrete structures at the second part of the 20^{th} century. Similar to the steel fibres, this reinforcement must be added into the concrete until it will be equally mixed. The average length of the fibres is from 40 mm up to 60 mm and their material is usually polymer (olefin, polypropylene etc.). The fibres can increase the residual flexural strength of the concrete. In the literature a considerable amount of publication can be found about how the synthetic fibres can increase the shear capacity of the concrete elements [2], [3]. The main areas of using synthetic fibres are the precast industry the industrial floors, the tunnels (shotcrete or TBM) and the tramlines- concrete railways.

Both of these synthetic reinforcements can increase the shear capacity of reinforced concrete elements, however there is no standard for their calculation method. In this article the formulas and recommendations will be presented which can help in the shear design of a synthetic reinforced concrete beams.

2 Shear in concrete structures

The shear stresses are special tension stresses with a perpendicular direcion to the principal compressive stress trajectories. Compressive trajectories in a simply supported beam can be seen in Fig 1.

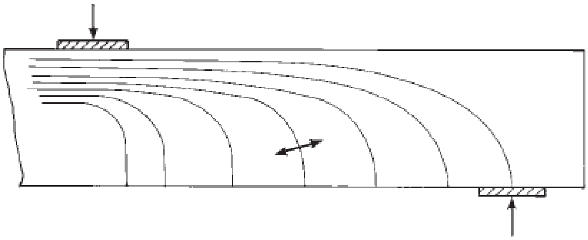

Fig 1 Principal compressive stress trajectories in a simple supported beam [4].

In a reinforced concrete beam the shear resistance comes from a contribution of several different effects. The first is the effect of the un-cracked compressed concrete zone. In reinforced concrete beams the depth of the compressive zone highly determines the shear resistance of the element. This part of the beam is un-cracked, therefore, some of the vertical forces can be transferred here.

In the tensile zone, shear forces transfer across a crack by mechanical interlock, when the shear displacement is parallel to the direction of the crack (Fig 2). This effect is called aggregate interlock. Huge amount of scientific research tried to determine the contribution of the aggregate interlock to the full shear resistance. Some researchers questioned the existence of the effect [5] and some of them determined the contribution can be even 50% [6]. The vast majority of the articles locate the contribution of aggregate interlock shear between 33 and 50 % but with increasing the crack width this value can be reduce [7].

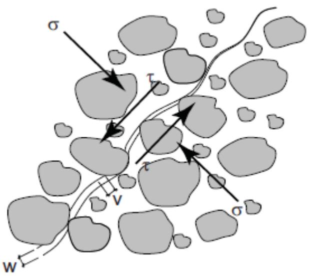

Fig 2 Mechanism of aggregate interlock [8] (w = crack width, v = parallel displacement of cracked edges, σ,τ = transferred stresses).

The dowel action is a combination of the tensile resistance of the concrete near to the flexural reinforcement and the bending and transverse shear resistance of the main reinforcement. According to the literature this shear component has the smallest contribution in the full shear resistance [9].

Shear links (stirrups), bent-up main reinforcement and fibre reinforcement can also take a contribution of a shear resistance of reinforced concrete beams. The bars bridge the two parts of the crack and can transfer the shear forces between the upper and the lower parts of the crack. The most efficient bars are perpendicular to the crack. The fibre reinforcement can also increase the shear resistance by bridging the cracks. The fibres with a randomly distribution can be effective independently of the place of the shear crack.

As it has been demonstrated above see the determination of the shear resistance requires a lot of attention, the contribution of the different effects can change in beams with different geometry, main or shear reinforcements. The current standards intend to simplify the shear mechanism, and summarize the different effects in a simple formula, which can be used for every reinforced concrete beam. One of the oldest explanations for reinforced concrete beams behaviour is the truss analogy. According to this analogy the behaviour of a simply supported concrete beams is similar to a truss system. The tension stresses are carried by the flange members and the shear stresses are beared out by the inclined compressed concrete trusses and by the optional shear reinforcement. This analogy is the basis of the formulas in several standards, which calculate the shear resistance for the concrete and for the shear reinforcement as well. Some standards define the angle of the concrete truss in a specific value (Eurocode) and some of them give the opportunity of the determination to the designer. However, in a reinforced concrete beam the shear capacity of the concrete and the shear capacity of the additional shear reinforcement can exist parallelly, the codes allow to use only one of them (the concrete shear resistance or the reinforcement's shear resistance), therefore, the formulas have significant safety.

3 Shear capacity determination for FRP reinforced concrete beams

The shear behaviour of the FRP reinforced concrete beams are close to the concrete beams with traditional steel bar reinforcement, because the failure mechanism is the same, just the material parameters of the FRP bars are different. However, this different material parameters and material behaviours can change the contribution some of the shear components. The material behaviour of FRP bars can be considered as perfectly linear-elastic: the stress-strain relationship of FRP bars is linear up to the failure there is no plastic part of the diagram. This means the material's failure can occur without any visible sign, which make the proper design necessary. The elastic modulus of these bars is from 70 000 MPa (glass) to 300 000 MPa (carbon). From the shear components the effect of the compressive concrete zone changes most significantly. In FRP reinforced concrete (FRP RC) beams the area of the compression zone after cracking is smaller than the traditional RC structures because of the low elastic modulus. However, in case of traditional RC structures the depth of the neutral axis decreased significantly after the yield of the steel bars. This phenomenon is not happening in case of FRP bars because of the material behaviour, the depth of the neutral axis is monotonically increasing after the first crack (Fig 3).

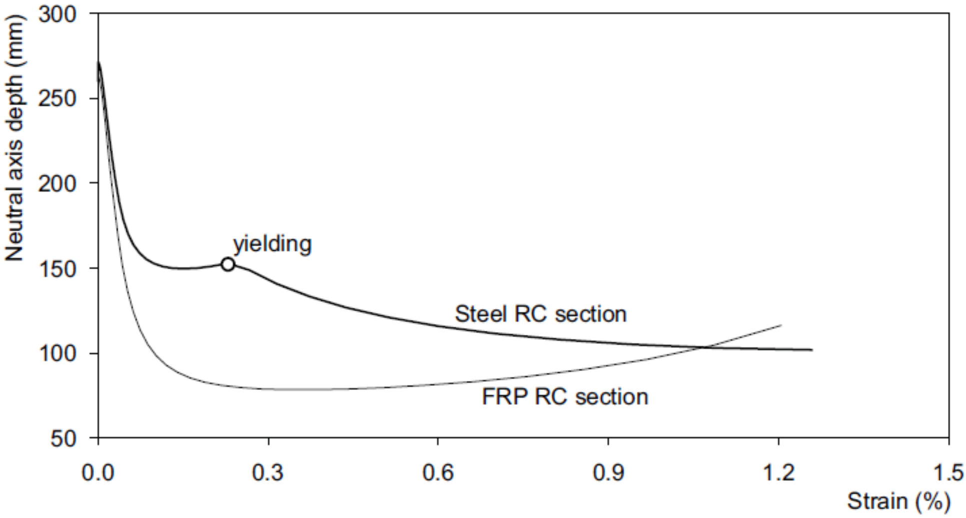

Fig 3 Depth of the neutral axis in terms of the strain of longitudinal reinforcements [10].

Because of the low elastic modulus the cracks are larger in case of concrete beams with FRP bars than RC structures in the same load level, the effect of the aggregate interlock is smaller. Also because of the FRP bars have a really low transversal stiffness, the dowel action is negligible [11]. The effect of the shear reinforcement depends on the tensile strength of the material which is usually the yield strength of the steel bars. In case of FRP the maximal elongation and the bond between the bar and the concrete is more significant because of the linear- elastic material behaviour. Because of this usually the standards use the strain limit for FRP bars: the strains in the bars must be under a defined value.

The standards and the guidelines contain separate chapters for shear design of FRP bar reinforced concrete structures. These formulas usually the modification of the formulas for traditional RC structures, with taking into consideration the effect of the low elastic moduli of the FRP reinforcement.

3.1 ACI 440.1R [12]

The formulas in the ACI recommendation are modifications for the shear formulas for RC structures according to the ACI 440.1R-15-2005 [12], but with using the maximum strain limit. The formula for concrete's shear resistance contains the effect of the FRP bars in the part of the calculating the depth of the compressive zone. The formula for concrete shear resistance with FRP main reinforcement is the following:

$$V_{cf} = 0.4\sqrt{f_c'}b_w c \tag{1}$$

where $c = k \cdot d$ for rectangular cross section

$$k = \sqrt{2\rho_f n_f + (\rho_f n_f)^2} - \rho_f n_f \tag{2}$$

where $\rho_f = \frac{A_f}{b_w d}$ and A_f is the area of the FRP reinforcement

In the presented equations the b_w is the width of the cross section, d is the effective depth, and n_f is the ratio of the elastic moduli of the FRP and the concrete.

The contribution of the FRP stirrups is similar to the steel ones, but defining the tensile strength of the FRP bar as 0.004 E_f. The code defines also the minimum reinforcement ratio for FRP stirrups, as 0.35/0.004 E_f.

3.2 Italian national research council [13]

The Italian design specification [13] recommends a formula as a modification of CNR 2006 national design code, which based on Eurocode 2. The contribution of the concrete is modified by taking into consideration the axial stiffness of the FRP bars as:

$$V_{cf} = 1.3 * \left(\frac{E_f}{E_s}\right)^{0.5} * \tau_{Rd} k(1.2 + 40\rho_f) b_w d \tag{3}$$

where ρ_f is the reinforcement ratio.

The contribution of the FRP stirrups is similar to the steel shear links, however there is a stress limit for the FRP bars, which is 50% of their design strength.

3.3 Design approach of Guadagnini et al. [14] (Modification of EN 1992-1)

This recommendation based on the Eurocode 2 [15] shear formula for calculating the shear resistance of the concrete with taking into consideration the ratio of the elastic modulus of the FRP and the steel bars.

$$V_{cf} = 0.12(1 + \sqrt{\frac{200}{d}})(100 \cdot \frac{A_f}{b_w d} \cdot \frac{E_f}{E_s} \cdot \phi_s \cdot f_{ck})^{1/3} b_w d \tag{4}$$

where Ef is the elastic modulus of the FRP, Es is the elastic modulus of the steel bars, fck is the characteristic value of the concrete's compressive strength. The ϕs represents the ratio of the maximum allowed strain in FRP and the yield strain of the steel bars.

The formula for FRP stirrups uses the strain limit as well, it defined by the maximum strain as 0.45%. The minimum reinforcement ratio ($\rho_{fw,min}$) for FRP stirrups can be calculated according Guadagnini et al. [14] as:

$$\rho_{fw,min} = 0.08\sqrt{f_c'} \cdot \frac{1}{0.0045 \cdot E_f} \tag{5}$$

4 Shear capacity determination for synthetic fibre reinforced concrete beams

The fibre reinforcement is a well-used material for shear strengthening, several recent studies show promising results with using steel fibre reinforcement (SFRC) as shear reinforcement [16]. The fibres increase the fracture energy of the concrete which makes the structure more ductile, and raise the residual flexural strength of the material. Because it was mentioned that the shear crack is a special type of the tensile cracks, the randomly distributed fibres can bridge the crack, and can transfer loads between the two parts. Also the fibre reinforcement decreases the crack width which helps to the aggregate interlock to be more efficient. The *fib* MC 2010 [10], the RILEM [17] recommendation and many literature gives design formulas for steel fibre reinforcement as shear reinforcement, but there is no guidelines for synthetic fibre reinforced concrete structures. However in the literature several recent articles show [2], [3], that synthetic fibre reinforcement (SynFRC) can be used as shear reinforcement as well. According to [18] the *fib* and the RILEM formulas can lead to proper results with SynFRC as well, and with these the synthetic fibre reinforced concrete shear resistance can be calculated.

4.1 *fib* MC2010 [10]

The Model Code 2010 [10] defines the fibre reinforced concrete beam with longitudinal reinforcement by adding the effect of the fibre reinforcement to the concrete's shear resistance:

$$V_{frc} = \frac{0.18}{\gamma_c}(1+\sqrt{\frac{200}{d}})(100\cdot\frac{A_f}{b_w d}\cdot(1+7.5\frac{f_{Ftuk}}{f_{ctk}})\cdot f_{ck})^{1/3} b_w d \tag{6}$$

where f_{ctk} is the characteristic value of the concrete's tensile strength

$$f_{Ftuk}(w_u) = f_{Fts} - \frac{w_u}{2.5}(0.5 * f_{R3} + 0.2 f_{R1}) \tag{7}$$

In Eq. 7 the w_u=1.5mm and the f_{Fts}=0.45f_{R1}. The f_{R1} and f_{R3} values are the residual tensile stress values at Crack Mouth Opening Distance (CMOD) stage 0.5 mm and 2.5 mm reprehensively. These values can measure from three point bending beam tests according to RILTEM TC 162 [17].

4.2 RILEM

The formula [19] was developed at the beginning of the 21th century to present a simple tool with a huge amount of safety for SFRC structures. During the years the formula modified, but the original one gives better correlation for synthetic fibre reinforced concrete beams.

The shear resistance of a SynFRC beam can be calculated by summarize the shear capacity of the concrete and the added shear resistance by the fibres.

$$V_c = 0.15\cdot\sqrt[3]{3(\frac{d}{a})}\cdot k(100\cdot\rho\cdot f_c')^{1/3} \tag{8}$$

$$V_{SYF} = \frac{1600-d}{1000}\cdot 0.5\frac{d}{a}f_{e,3} \tag{9}$$

$$V_{frc} = V_c + V_{SYF} \tag{10}$$

where a is the shear span, ρ is the reinforcement ratio, $f_{e,3}$ is the equivalent flexural strength.

5 Shear capacity determination for synthetic fibre reinforced concrete beams with longitudinal FRP bars

The combination of using FRP bars as longitudinal reinforcement and synthetic fibre reinforcement as shear reinforcement can be an alternative solution for traditional reinforced concrete where the conditions require the non-corrosively of the reinforcement. In the standards, guidelines or codes are not a design formula for these structures. However, according to the mentioned literature the shear resistance of a synthetic fibre reinforced concrete beam with longitudinal FRP reinforcement and without stirrups can be estimate as follows:

$$V_{frc,f} = \frac{0.18}{\gamma_c}(1+\sqrt{\frac{200}{d}})(100\cdot\frac{A_f}{b_w d}\cdot\frac{E_f}{E_s}\cdot\phi_s\cdot(1+7.5\frac{f_{Ftuk}}{f_{ctk}})\cdot f_{ck})^{1/3} b_w d \tag{11}$$

This equation should be verified by laboratory tests thereafter it can be a good opportunity to predict analytically the shear resistance of a non- corrosive reinforced concrete beam. The formula takes into consideration the ratio of the FRP and steel material, and the additional shear capacity from synthetic fibre reinforced concrete as well. The base of Eq. 11 is the shear resistance formula for FRP reinforced concrete beams according to Guadagnini at all [14] and the *fib* MC2010 [10] shear resistance formula for steel fibre reinforced concrete beams.

6 Numerical verification of the analytical model

In 2015 an FRP reinforced concrete beam test serie was carried out at Budapest University of Technology and Economics [3]. The geometry of the beams was the following: 1100 m long, 160 mm high and 100 mm wide. The test was a simple three point bending beam test with a span of 1.0 m. The beams were reinforced with 2 pieces of 6 mm diameter longitudinal tensioned FRP bars made from basalt and glass as longitudinal tensile reinforcement. The mean material parameters of the FRP reinforcement can be seen in Table 1. For each type of FRP reinforcement 3 samples were casted. The beams did not contain any stirrups, but they were made with 5.0 kg/m3 synthetic fibre reinforcement. The material parameters of the fibres and the residual strength values can be seen in Table 2. The residual strength values were determined according to the RILEM TC 162 [17].The concrete strength class was C50/60 accoring to Eurocode 2. [15] and the nominal concrete cover was 10 mm.

Table 1 Material parameters of the FRP bars.

Material of the FRP bar	Sign	Glass (GFRP)	Basalt (BFRP)
Elastic modulus	E	70 GPa	45 GPa
Tension strength	f_t	1332 MPa	1100 MPa
Maximum strain	ε	2.44 %	2.2 %

Table 2 Properties of the synthetic fibres.

Material of the fibres	Sign	Modified olefin
Elastic modulus	E	12 GPa
Tensile strength	f_t	640 MPa
Residual strength values with 5.0 kg/m3 dosage in C50/60 concrete	f_{R1}	2.45 MPa
	f_{R2}	3.05 MPa
	f_{R3}	3.30 MPa
	f_{R4}	3.05 MPa

However the added synthetic fibres increased the shear capacity of the FRP reinforced concrete beams, the falier mode was in all case shear failure. The verification of the Eq. 11 was done with using the presented parameters for FRP bars and for fibres and the measured mean values for concrete parameters. To be able to compare the calculated and the test results no safety factor was used during the calculation.

The mean results of the laboratory test and the calculated shear capacities can be seen in Table 3.

Table 3 Results of the verification.

	Laboratory	Calculated	Safety (lab/calc)
GFRP + 5.0 kg/m^3 synthetic fibres	28.78 kN	17.88 kN	1.61
BFRP + 5.0 kg/m^3 synthetic fibres	22.61 kN	15.58 kN	1.45

The results show that the formula represents well the real shear capacity of the FRP bars reinforced synthetic fibre reinforced concrete beams, the calculated and the measured shear capacities are really close to each other. These formulas contain a relative large safety factors also with using steel reinforcements [20], this is why in case of synthetic reinforcements there is also safety even using mean material parameters. According to the results the developed analytical model can be used for design the synthetic material reinforced concrete beams with safety in mind.

7 Conclusion

The shear failure of reinforced concrete beams is one of the most complex failure modes, where inclined crack disconnect the upper and the lower part of the beam. The shear resistance of reinforced concrete beams depends on different effects such as effect of the compressed concrete zone the aggregate interlock, the dowel effect and the effect of the shear reinforcement. The contribution of these effects depends on many parameters, such as beam geometry, reinforcement ratio type of shear reinforcement, and also has an impact on each other.

Standards, codes and guidelines specify the shear resistance for steel reinforced concrete beams and steel fibre reinforced concrete beams as well. For non- corrosive materials, such as FRP bars or synthetic fibre reinforced concrete elements the literature recommend formulas to calculate the shear resistance. These formulas are basically the modification of the shear capacity equations for traditional steel reinforced concrete.

With merging the different formulas for FRP reinforced concrete beams and for SynFRC beams the shear capacity of the synthetic fibre reinforced concrete beam with longitudinal FRP reinforcement can be estimate according to Eq 11. The formula summarizes the effect of the FRP bars to the concrete's shear resistance, and the additional shear capacity of synthetic fibre reinforcement. The verification of the formula was done with using laboratory test results. The results shows that the formula represents well the real shear capacity of the FRP bars reinforced synthetic fibre reinforced concrete beams. The developed analytical model can be used by the engineers to determine the shear capacity of synthetic reinforced beams, however further verification of the formula is necessary with using different type of FRP bars and synthetic fibre reinforcement.

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