# FIBRE REINFORCED CONCRETE CALCULATIONS IN ULTIMATE AND SERVICEABILITY LIMIT STATE

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## SUMMARY

Macro fibres have been well accepted for the reinforcement of sprayed concrete in mining and tunnelling applications, and today are starting to be used in other structural elements either in combination with existing steel bars, reducing the size of the steel bars or replacing the steel altogether. Although the use of fibre-reinforced concrete in the building industry is already common practice, generally accepted design methods are still lacking. Due to this many engineers are hesitant to use fibre-reinforced concrete. This paper describes a calculation method for the use of fibre-reinforced concrete for the ultimate limit state (ULS) and serviceability limit state (SLS) using the modification of the latest Austrian guideline - Österreichusche Vereinigung für Beton- und Bautechnik 2008 (OVBB).

## 1. INTRODUCTION

FRC beams reinforced with a low-dose of macro fibres show a reduction in the load-displacement diagram which means that the moment after first crack is less than before cracking in the elastic situation. This indicates that the failure will occur in statically determinate structures (e.g. simple supported beam) when it reaches and exceeds the elastic moment limit. For this reason low-dose FRC is used for statically indeterminate secondary structures (e.g. industrial floors), or primary structures if the risk to human life is small or economic consequences are either not significant or negligible (e.g. some house bases, cellar walls, pools). FRC can only be used for statically determinate primary structures if a high-dose of fibre or a combination of fibre and steel bars is used. The fibres could reduce the area of the reinforcement, the crack width and the deflection of the structure.

## 2. CALCULATION METHODS

The designs of RC structures are calculated in three stress states: (i) elastic, uncracked, (ii) elastic, cracked; and (iii) plastic (fig. 1). (i) and (ii) are used for the serviceability limit state and (iii) for the ultimate limit state. The serviceability requirements of the structure are determined by the crack width and the deflection criteria of the concrete.

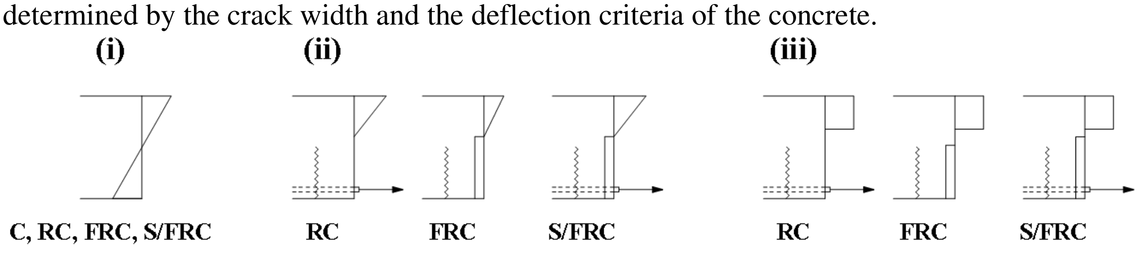


Figure 1: stress states

## 3. MATERIALS

The effect of the fibre to the FRC, S/FRC highly depends on the dosage, shape and material of the fibre. There is a minimum dosage of fibre under which there is no effect on performance (similar to a minimum steel ratio). The maximum fibre dosage is determined by the ability of the fibres to mix with the concrete. The paper focuses on a low dosage (0,1-2 V%) FRC (Balázs, 1999).

FRC is a cement-based composite material reinforced with randomly distributed, short fibres. The main benefit of the fibre is the post-cracking properties i.e. concrete becomes a more ductile material. Development of the post-crack stress is due to the ability of the fibres to transfer tensile stress across a cracked section, potentially leading to a reduction in crack widths.

A modified version of the Austrian guideline (OBVV) has been used to calculate the FRC beam. The stress-strain diagram was simplified for hand-calculations (L), non-linear calculations (NL), which are presented in the guideline. The calculation with diagram (L) is easy to use, however gives less accurate results. The (NL) diagram is not suitable for hand calculations. I recommend the (NLM) diagram which gives a better result than (L), presumably closer to reality, but still could be used for hand calculations (fig 2.).

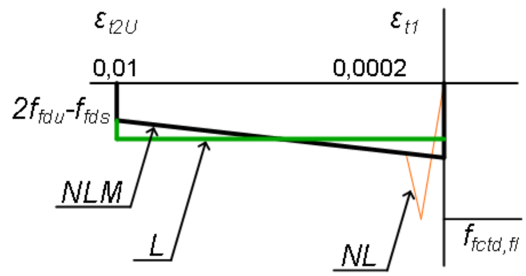


Figure 2: stress-strain diagrams for FRC

The steel reinforcement elastic-plastic diagram is as recommended by the Eurocode (EC) and only the tensioned bars are taken into consideration.

**4. THE CALCULATIONS**

In the elastic non-cracked stress state (i) the effect of the macro fibres, as for other reinforcement, is negligible. We need to use this state for the deflection calculations.

We need to take into consideration further conditions at the (ii) stress state. Concrete as well as FRC could be at the maximum strain condition when we are at the limit of the stress state, and the steel could be elastic or plastic. The moment-curvature diagram has been calculated from the formulas (fig. 3). This is the basis for the maximum moment and deflection calculations.

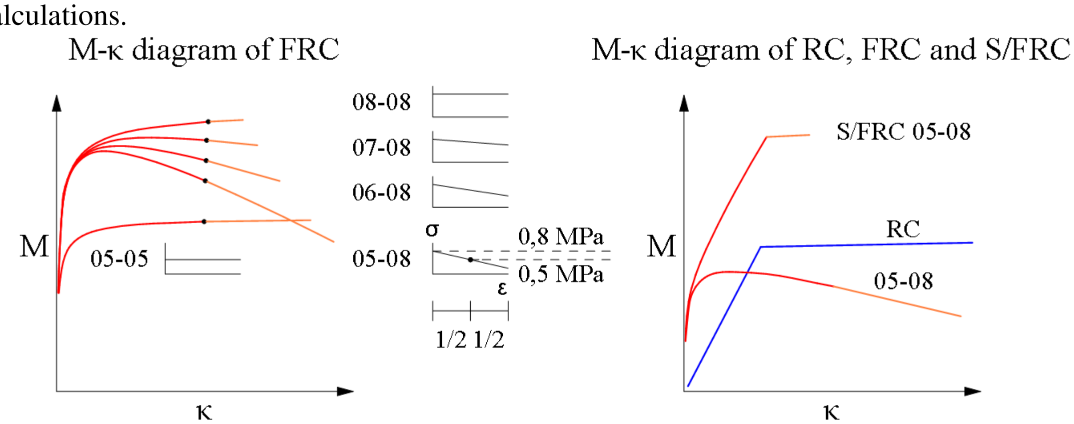


Figure 3: moment-curvature diagram from different stress-strain relationships

## 5. ULTIMATE LIMIT STATE

The moment-slope diagram shows the maximum moment for the ultimate limit state which could be calculated in the (ii) stress state for FRC and (ii) or (iii) stress state for S/FRC which depends on the steel/concrete ratio. Only the moment capacity calculation was taken into account.

## 6. SERVICEABILITY LIMIT STATE

### 6.1. Deflection

Deflection of RC structures is calculated from the curvature, so the moment-curvature relationship is required. This relationship is linear in the (ii) stress state for RC and non-linear for FRC and S/FRC.

The formula for curvature in EC is as follows (taking into consideration the „tension-stiffening"):

$$\kappa = (1-\varsigma)\kappa_I + \varsigma\,\kappa_{II} \quad (1)$$

The deflection is the double integral of the curvature.

If we change the curvature of RC to the curvature of S/FRC we get the following function that is used for the calculation of deflection.

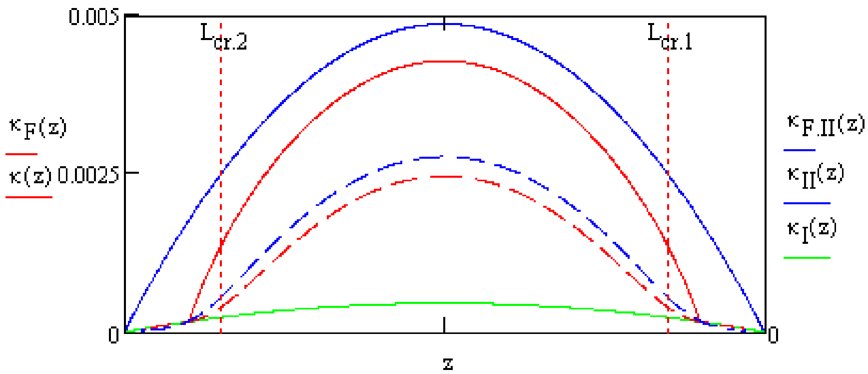


Figure 4: function of curvature of a two support beam
blue line: curvature according to (ii) stress state, green line: curvature according to (i) stress state, red line: curvature for deflection calculations, solid line: RC, dashed line: S/FRC

According to the improved calculation the displacement are reduced, however we have to take into consideration the following:

a) The steel is in elastic state, but the FRC in „plastic". For this reason I recommend to calculate the loads with the frequent combination instead of quasi-permanent combination.
b) Further research is needed to determine how the fibre modifies the tension-stiffening.
c) Research on the creep of FRC continues.

## 6.2. Crack control

The EC gives the following formula for the calculation of crack widths:

$$w_k = \beta s_{rm} \varepsilon_{sm} \text{ [mm]} \tag{2}$$

where $s_{rm}$ is the distance of the cracks. The Italian guideline for FRC CNR-DT 204/2006 (2006), changes only the calculation of $s_{rm}$ with $\xi$. This dimensionless coefficient takes into consideration the fibres by their diameter and length ratio, but not the dosage. This coefficient could reduce the $s_{rm}$ by upto 50%.

The RILEM 162-TDF gives a calculation method for both FRC and S/FRC. The recommendation for S/FRC is very simple - that a reduction of the stress in the steel bar by the fibres has to be taken into consideration for the EC calculation. It gives a simple and easy to use formula:

$$w = \varepsilon_{fc,t} (h - x) \tag{3}$$

## 7. CONCLUSION

A FRC section has the biggest moment in (ii) stress state, so both the ULS and SLS must be verified here. Because of this FRC can be used economically in situations where both the ULS and SLS have a similar value, for example a quasi-permanent value is high and the safety factor is low, the load is permanent (e.g. industrial floor, hydrostatic pressure tanks).

As for S/FRC the fibre will reduce the deflection, the amount of steel reinforcing area required and the crack width. Where steel reinforcing is designed for crack control or the reduction of deflection and not for load bearing reasons the fibre could succeed (e.g. watertight products).

## 9. REFERENCES

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