# FIBRE REINFORCED CONCRETE CONSTITUTIVE LAWS FOR NUMERICAL SIMULATION

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## SUMMARY

Determination of appropriate material parameters for the fibre reinforced concrete material model in design and assessment of structures is an important task, which is necessary for realistic modelling of FRC structures. Various recommendations and definitions are specified by specialized groups as RILEM, *fib* or ACI. These recommendations deal with test results of beam submitted to three or four point bending load with or without notch. Authors compared these documents and applied the determined parameters in numerical simulations. As the achieved results were not very satisfactory, a new solution for definition of the FRC parameters should be found. Presented solutions involve inverse analysis of results and method proposed by Juhász (Juhász, 2013). This methodology applies so-called modified fracture energy: the fracture energy of concrete is extended by energy contribution related to fibres.

## 1. INTRODUCTION

There are many guidelines to model and design fibre reinforced concrete, such as RILEM TC162-TDF (Vandewalle et al., 2003), *fib* Model Code 2010 (*fib* Model Code, 2010), ÖVBB Richtlinie Faserbeton (Richtlinie Faserbeton, 2008), ACI 544.8R-16 (ACI 544.8R-16, 2016), CNR DT (CNR-DT 204/2006, 2007). All of these guidelines are based on a three or four point bending tests. Obtained test results as load-displacement or load-CMOD diagrams are converted to the parameters that can be used as a material model. RILEM and *fib* Model Code 2010 use residual strengths and define "stress-crack width" or "stress-strain" diagrams that can be applied as material laws. ACI describes an indirect method to obtain the stress-strain response.

In finite element model the stress-strain diagram can be used if the characteristic length and the direction of the principal stress is known. In this case, the stress-strain can be converted to stress-crack width and the crack localization can be handled. For this purpose, the crack band size method can be used (Bazant, 1984).

These stress-strain models based on guidelines can be applied if the Bernoulli-Navier hypothesis is valid (and as a consequence there is a linear elastic stress distribution in the cross section). However, in reality, the stress distribution will be different due to the notch that is usually in the middle of the specimen for three point bending test or due to the cracks in the material that are formed during the test.

In this case, it is necessary to obtain material laws for finite element analysis by different method. As was shown in previous papers (Pukl et al., 2013), (Sajdlová & Pukl, 2014) material parameters can be determined by inverse analysis of results from basic tests as three of four point bending. Another method was proposed by Juhász (Juhász, 2013) and it works with fracture energy of material composed of concrete matrix and fibres. It is reasonable to model the FRC with "stress-crack width", instead of "stress-strain" diagram. The shape of the softening curve after post crack can depend on the type of the fibre and the dosage, but most of the time it can be simplified as a constant value after crack (so called residual strength). The area under the "stress-crack width" diagram is the fracture energy. This fracture energy could be divided into 2 parts: fracture energy of the concrete matrix $G_F$ and added fracture energy of fibres $G_{Ff}$, see Fig. 1. These two methods are discussed in this paper and compared with material laws obtained by guidelines.

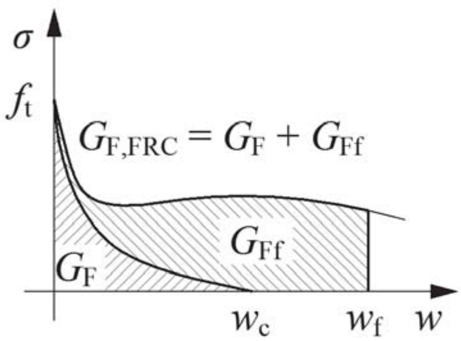
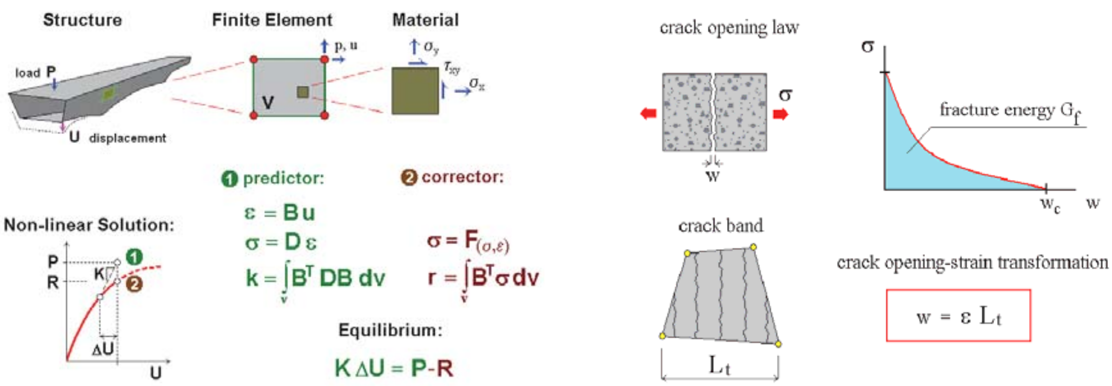



Fig. 1: Fracture energy of the concrete and added fracture energy by the fibres

## 2. FINITE ELEMENT ANALYSIS

Behaviour of FRC material is analysed in program ATENA (Cervenka et al., 2016) for non-linear analysis of concrete structures. ATENA is capable of a realistic simulation of concrete behaviour in the entire loading range with ductile as well as brittle failure modes as shown for instance in (Cervenka, 2002). It is based on the finite element method and non-linear material models for concrete, reinforcement and their interaction. The tensile behaviour of concrete is described by smeared cracks, crack band and fracture energy and the compressive behaviour of concrete by a plasticity model with hardening and softening. The constitutive model is described in detail in (Cervenka & Papanikolaou, 2008). Nonlinear solution is performed incrementally with equilibrium iterations in each load step.

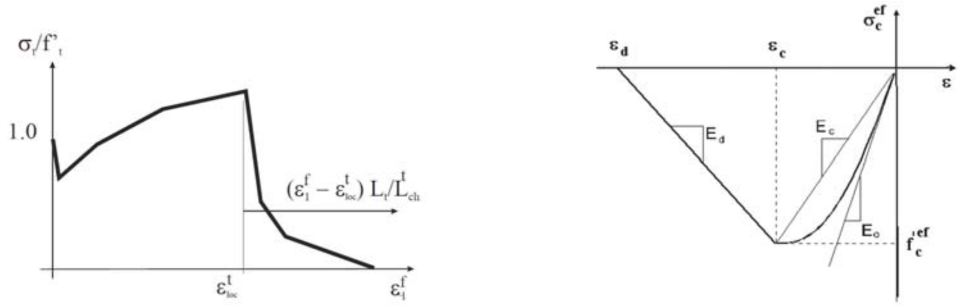



(a) (b)

Fig. 2: (a) Scheme of the nonlinear finite element method, (b) smeared crack model for tensile behaviour of concrete

### 2.1 FRC Material Models

The tensile response of FRC differs from normal concrete not only in the values like tensile strength and especially fracture energy, but also in the shape of tensile softening branch. The original exponential function valid for normal concrete can be used as a first approach, but preferably would be to use more realistic form of the tensile constitutive law. Therefore, special material models at macroscopic level are needed for modelling of fibre reinforced concrete.




(a) (b)

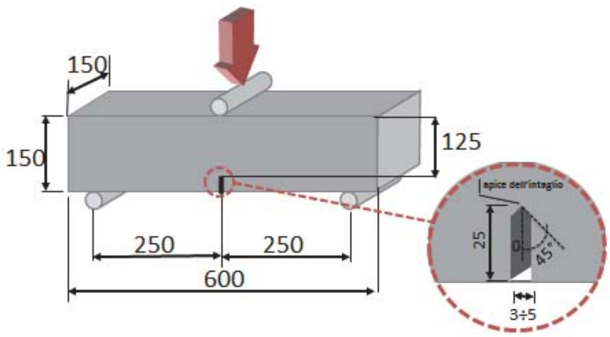
Fig. 3: (a) User defined tensile behaviour, (b) compressive stress-strain law

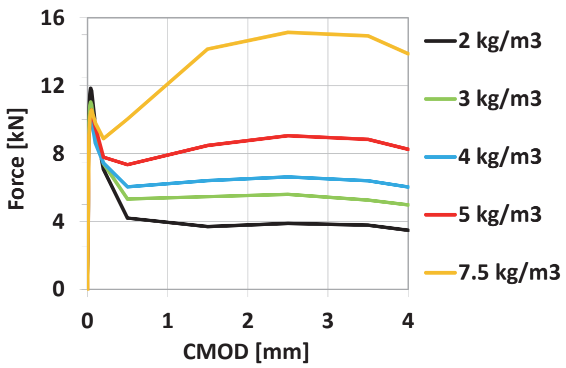
The most sophisticated and most general model of FRC material represents an extension to the fracture-plastic constitutive law (Cervenka & Papanikolaou, 2008) called 3D NLC2 User model. It describes the tensile behaviour according to the material response measured in tests point-wise in terms of the stress-strain relationship. The first part of the diagram is the usual stress-strain constitutive law. After exceeding the localization strain $\varepsilon_{loc}$ the material law assumed for the characteristic crack band width $L_{ch}$ is adjusted to the actual crack band width $L_t$. The characteristic crack band width (characteristic length) is the size (length) for which the defined material law is valid. The same procedure (with eventually different characteristic length) is used for the compression part of the material law. Compressive stress-strain law of mentioned material models is described in Fig. 3. The softening law in compression is linearly descending and the end point of the softening curve is defined by plastic displacement $w_d$ (corresponding to $\varepsilon_d$ in Fig. 3b). By increasing material parameter $w_d$ the contribution of the fibres to the compressive behaviour of concrete is considered. Another important compressive parameter for FRC modelling is reduction of compressive strength due to cracks which says how the strength is reduced while the material is subjected to lateral tension.

### 3. EXPERIMENTAL PROGRAM

Experimental program focused on application of synthetic fibres called BarChip48 in concrete C25/30 is chosen for the presented study. Different dosages of fibres were tested as is shown in load-displacement diagrams in Fig. 4b. Six tests were provided for each dosage, the plotted curves represent mean values. Geometry of the specimen and test setup corresponds to EN 14651 (EN 14651, 2005). Beams were tested under three point bending condition. The cross section is 150x150 mm and span is 500 mm. The central part of the beam is weakened by notch 25 mm long, see Fig. 4a.

Result for fibre dosage 2 $kg/m^3$ was chosen for numerical analysis presented in this paper.




(a) (b)

Fig. 4: (a) Geometry of tested specimen, (b) comparison of LD diagrams for different fibre dosages

## 4. MATERIAL LAW FOR FIBRE REINFORCED CONCRETE

### 4.1 Recommendations from guidelines

As a representative document, RILEM TC162-TDF is chosen for determination of the material law. Experimental programme described in previous chapter involves the same test procedure and specimen geometry that is described in RILEM. Flexural tensile strengths $f_{R,i}$ are determined Based on Bernoulli-Navier hypothesis by expression:

$$f_{R,i} = \frac{3F_{R,i}L}{2bh_{sp}^2} \tag{1}$$

where $b$ is width of the specimen,
$h_{sp}$ is distance between tip of the notch and top of cross section,
$L$ is span of the specimen,
$F_{R,i}$ is load recorded at $CMOD_i$.

Maximal flexural strength $f_{fctm,fl}$ and residual flexural strengths for crack mouth opening displacement *CMOD* 0.5 mm ($f_{R,1}$) and 3.5 mm ($f_{R,4}$) are used for the determination of material law, see Fig. 5 left. Stress-strain diagram defined by RILEM is trilinear, for the numerical model part after the peak is important. Final diagram utilized in the numerical analysis contains bilinear softening as is shown in Fig. 5 right.

The finite element model for bending test is made for a plane stress simplification, with low order quadrilateral elements with 2x2 integration scheme, with the square elements shape and size of 5 mm, i.e. 30 elements through the height (25 elements above notch), see Fig. 6a. The loading is applied by force on the top loading plate. *CMOD* is calculated as difference between horizontal displacement of the right and left bottom part of the notch. Characteristic length for tensile stress-strain diagram is equal to the element size, i.e. 5 mm.

Comparison of the load-displacement diagram from test and numerical simulation is shown in Fig. 6b. Model can correctly describe behavior on the tail of the diagram but there are differences after the crack localization. Model according to RILEM underestimate the flexural strength of the material.

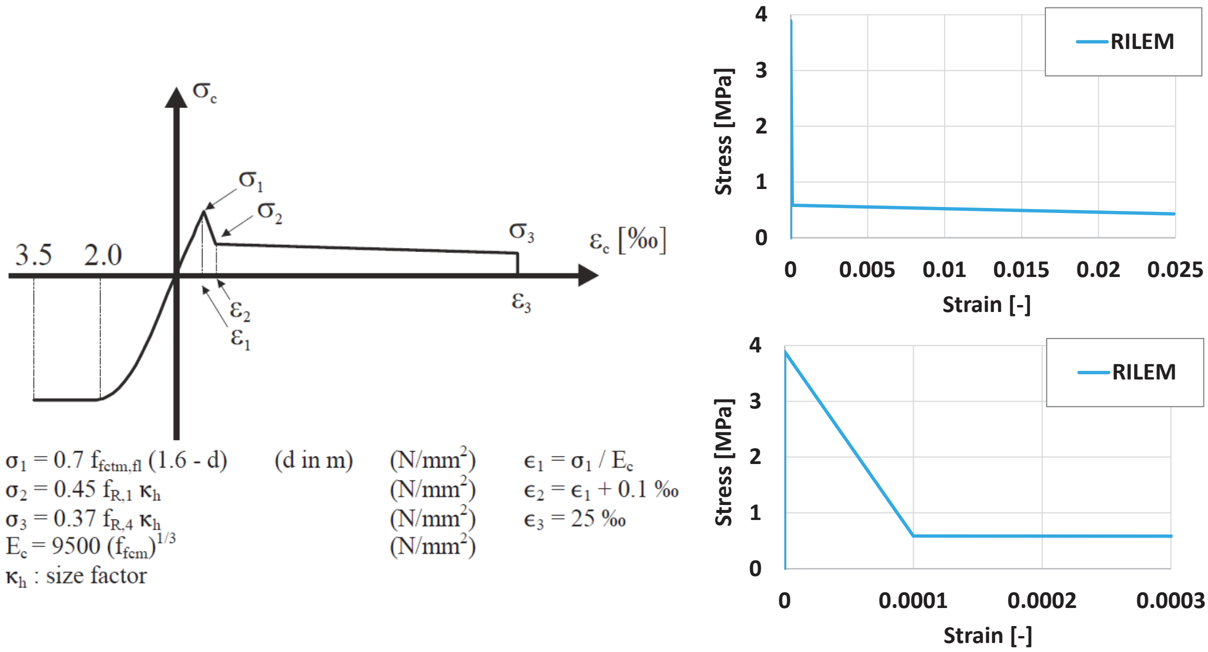
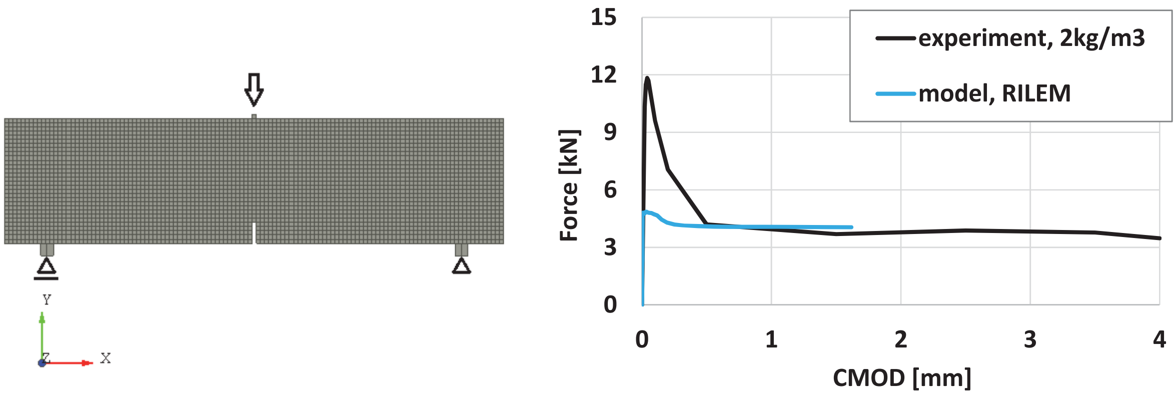



Fig. 5: (left) Stress-strain diagram according to RILEM TC162-TDF (Vandewalle et al., 2003), (right top) diagram for material with fibre dosage 2 kg/m$^3$, (right bottom) detail of the diagram until strain 0.0003




(a) (b)

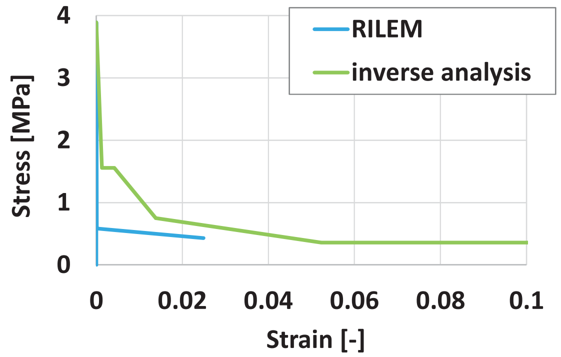
Fig. 6: (a) FEM model of three point bending test, (b) comparison of experimental result for fibre dosage 2 kg/m$^3$ and model with RILEM material law (characteristic length 5 mm)

### 4.2 Inverse analysis

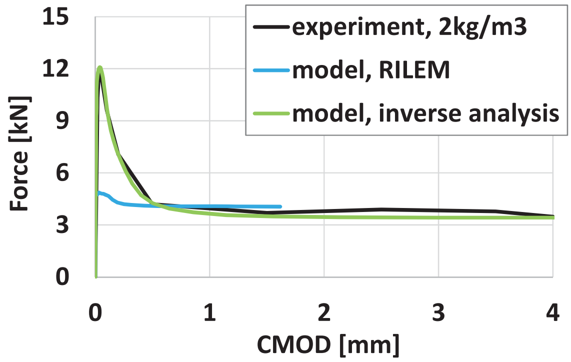
Another way how to obtain FRC material law for nonlinear finite element analysis is inverse analysis of experimental results which consists of two main steps. The first one is the estimation of material law based on mixture, contents and type of fibres, etc. or guidelines recommendations. The second step is modification of initial law by inverse analysis of material tests, mainly four-point bending test until the required accuracy of results is achieved.

In this case, RILEM material law was utilized as an initial function and by several simulations it was modified to the optimal material law for investigated FRC that is shown in Fig. 7a. It is obvious that material law is described in more detail compared to the RILEM and it leads to more accurate behaviour during bending test, see Fig. 7b.

Advantage of this approach is that it can be used for any experimental result and specimen geometry and it is possible to describe material very precisely. As a disadvantage, more than one numerical simulation are necessary for satisfactory result. For example, presented result was found by performing three analyses.




(a)




(b)

Fig. 7: (a) Stress-strain from inverse analysis in comparison with RILEM, (b) comparison of experimental result for fibre dosage 2 kg/m$^3$ and model with RILEM material law and law obtained by inverse analysis

### 4.3 Modified fracture energy method

The third method was proposed by Juhász (Juhász, 2013) and it utilizes the fracture energy of the FRC.

A thin band with micro-cracks will appear due to the tensile stress in the concrete – which is called the crack process zone. By increasing the stress the concrete reaches its tensile strength when the micro cracks are touching each other. After this point the tensile capacity of the concrete will decrease, the cracks will bypass or cross the aggregates and then the entire section will be crossed by the crack. The area under the "tensile stress – crack width" diagram is the fracture energy.

The fracture energy of the concrete is influenced by a number of factors which are clearly not related to the concrete's strength class. Most of the existing design methods neglect the fracture energy of the concrete and do not pay much attention to the tensile strength. However, when designing FRC structures these parameters cannot be ignored.

The main goal in this method is to separate the fracture energy of the concrete ($G_F$) and added fracture energy by the fibres ($G_{Ff}$). According to previous research (Juhász, 2015), the added fracture energy depends on the fibre type, dosage and cement mortar (cement, water and sand). By knowing these values the added fracture energy could be defined and used as a parameter partly independent from the concrete. In this research, concrete with the same cement mortar but with different aggregate type and size was made. In the case of normal aggregate (type *A*, *B* and *C*) the added fracture energy ($G_{Ff}$) was mostly unchanged.

Application of this method in the numerical modelling to compare result with methods mentioned in the previous chapters will be provided during the further research.

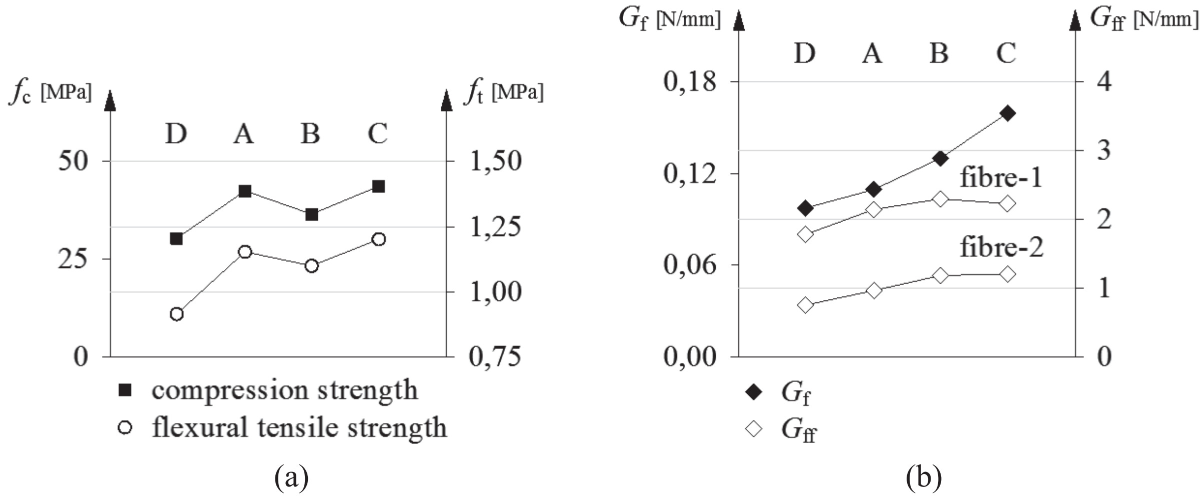



Fig. 8: (a) Compression strength and flexural tensile strength of concrete A, B, C and D, (b) fracture energy of the concrete A, B, C, D and added fracture energy of fibre-1 and fibre-2

## 5. CONCLUDING REMARKS

A study utilizing different methodologies for determining FRC material parameters and consequent numerical analyses were performed. As the recommendations from guidelines for the FRC tensile material law are not sufficient for numerical simulations, it is necessary to find another options. The first one is inverse analysis of results that is general and can be applied to any experiment. However, this procedure can be time-consuming because it requires several simulations. The next goal is to create tool for automatic inverse analysis of FRC material parameters in software ATENA. Another option is modified fracture energy method proposed by Juhász (Juhász, 2013). Knowing or estimating the added fracture energy could be a useful parameter to the FRC model, however it depends on the concrete mortar. Further research is required to determine the degree of impact of the cement grout alteration on the added fracture energy and also to find relation with the dosage of the fibres. The advantage of the model is that the fibre reinforced concrete can be described by one parameter.

## 6. ACKNOWLEDGEMENTS

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